Steady state Heat conduction:-

→ In ~~Heat~~ Heat transfer Analysis, we often interested in the rates of Heat transfer through Medium under steady conditions and surface temperature.

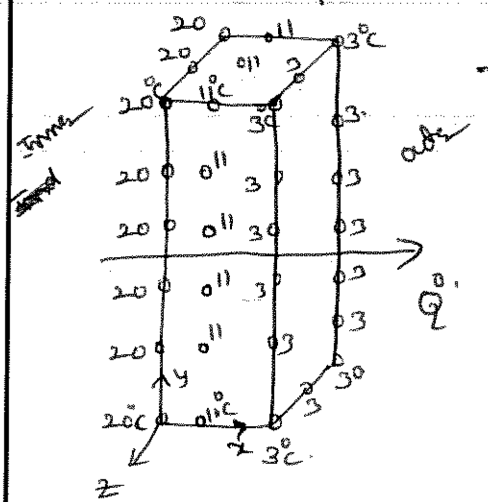
→ Such problems can be solved easily without involving any differential equations by the introduction of thermal Resistance concept in an analogous manner to electrical circuit problems.

→ Thermal Resistance corresponds to electrical resistance, temp difference corresponds to voltage, and heat transfer rate corresponds to electrical current.

→ We start in this chapter with one-Dimensional Heat conduction in a plane wall, a cylinder and a sphere and develop thermal Resistances in these geometries.

→ We also discuss the thermal contact resistance and the overall heat transfer co-efficient.

Example of 1-D steady state:-



→ In this figure the Heat Lost continuously to outer through wall & it is in the normal direction.

→ It is certain direction is driven by temp gradient (dT/dx).

→ The temperature at any Location is Isothermal & there is no temp change from Left face to Right face and Top to Bottom, the temp difference is in only b/w Inner and outer surface

→ In this chapter we discuss about two different conditions of steady state Heat conduction in One Dimensional i.e.,
1) steady state Heat conduction in 1-D with out Heat (q_g). generation
2) " " " " " " with Heat generation

→ For steady state 1-D with out q_g we ~~is~~ oftenly used thermal Resistance concept, & it is like Electrical ~~And~~ Analogy.

→ The Governing Differential equation for solid with out Heat generation (q_g) in steady state 1-D distribution is

$$\boxed{\frac{d}{dX}\left[X^n \cdot \frac{dT(X)}{dX}\right] = 0}$$

where

$X = x$ & $n = 0$ → For Rectangular coordinates

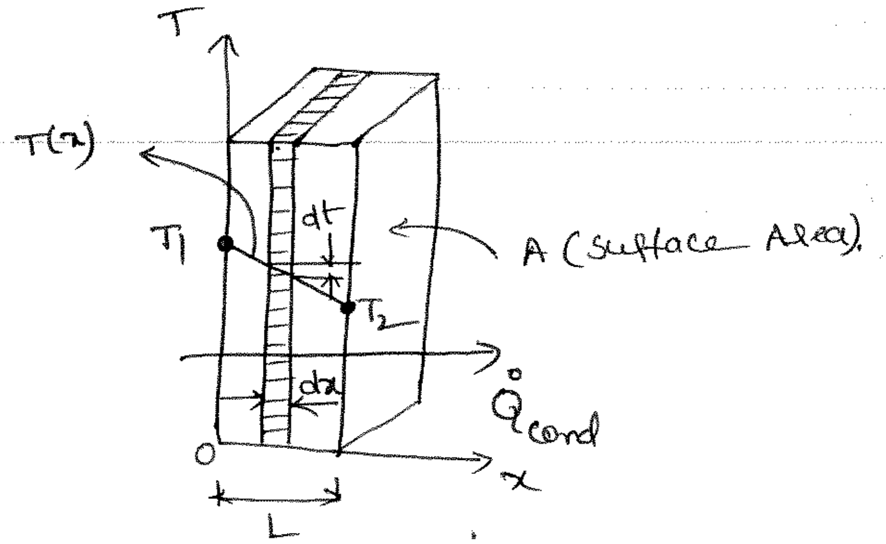
$X = r$ & $n = 1$ → " cylindrical "

$X = r$ & $n = 2$ → " spherical "

→ consider A plane wall of thickness L, ~~and~~ Average thermal conductivity K & The two surfaces are ~~Maintained~~ maintained at a constant Temperature's T_1 & T_2.
For steady state 1-D Heat conduction through the wall $T(x)$, then the Fourier's law of Heat conduction for wall is

$$\boxed{\dot{Q}_{cond} = -KA\frac{dT}{dx}} \quad \text{— watts}$$

$\frac{dT}{dx}$ = constant and The temp through the wall varies linearly with $\underline{x}$ & it is straight line



→ Now we Discuss steady state with out q_g later with q_g for all coordinates for 1-Dimensional.

1) steady state one Dimensional Heat conduction in a plane slab with out Heat Generation :-

Assumptions :-

1) steady state
2) one Dimensional
3) No Heat generation.
4) constant Thermal conductivity [K = const]
5) Material is Homogenious.

∴ The Fundamental Equation for cartesian coordinates in 3-D is

$$\Rightarrow \frac{\partial^2 T}{\partial x^2} + \underbrace{\cancel{\frac{\partial^2 T}{\partial y^2}}}_{1-D} + \underbrace{\cancel{\frac{\partial^2 T}{\partial z^2}}}_{1-D} + \underbrace{\cancel{\frac{q_g}{k}}}_{\text{No } q_g} = \underbrace{\frac{1}{\alpha}\cancel{\frac{\partial T}{\partial t}}}_{\text{Steady state}}.$$

$$\Rightarrow \frac{\partial^2 T}{\partial x^2} = 0$$

$$\Rightarrow \frac{d^2 T}{dx^2} = 0.$$

Integrating

$$\frac{dT}{dx} = C_1$$

Again Integrating

$$\boxed{T = C_1 x + C_2}$$

Now using First Boundary conditions

At, $x = 0 \Rightarrow T = T_1 \rightarrow$ left face

$$T = C_1 x + C_2$$

$$T_1 = C_1(0x) + C_2$$

$$\boxed{T_1 = C_2}$$

Now using secondary Boundary conditions

At, $x = L$, $T = T_2 \rightarrow$ Right face

$$T = C_1 x + C_2$$

$$T_2 = C_1(L) + T_1 \qquad [\because T_1 = C_2]$$

$$C_1 L = T_2 - T_1$$

$$\boxed{C_1 = \frac{T_2 - T_1}{L}}$$

$$\therefore \quad T = C_1 x + C_2$$

$$\boxed{T = \left(\frac{T_2 - T_1}{L}\right) x + T_1}$$

The Heat Conduction :-

$$Q = -KA \cdot \frac{dT}{dx}$$

$$Q = -KA \cdot C_1$$

$$Q = -KA \frac{(T_2 - T_1)}{L}$$

$$\boxed{Q = \frac{KA (T_1 - T_2)}{L}}$$

Electrical Analogy (Concept of Thermal Resistance):-

→ Resistance is nothing but Against of conduction, & simply we called as conduction Resistance

→ Conduction Resistance may happen in both electrical circuits and Heat circuits

→ According to Electrical Analogy.

$$\boxed{\Delta V = I R_{Electrical}}$$

→ like wise According to Thermal Resistance

$$\boxed{\Delta T = Q R_{th}}$$

→ According to Heat conduction in plane wall

$$Q = \frac{KA(T_1 - T_2)}{L}$$

$$Q = \frac{KA\Delta T}{L}$$

$$\Delta T = Q\left[\frac{L}{KA}\right] \qquad [\because \text{Like } \Delta T = Q R_{th}]$$

$$\boxed{\Delta T = Q[R_{th}]}$$

$\therefore$ R_{th} = Thermal Resistance in conduction

$$= \frac{L}{KA}.$$

Units of R_{th} :– $\frac{m}{\frac{W}{mK} \times m^2} = \frac{m \times mK}{W m^2} =$

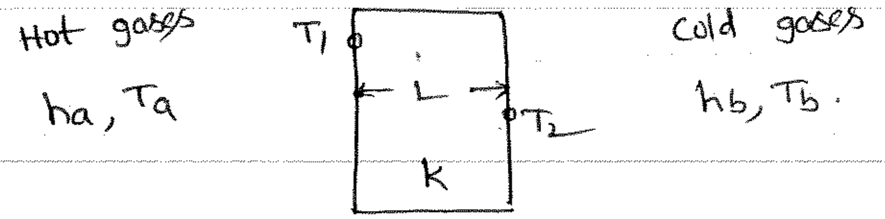
$$= \boxed{\frac{K}{W} \text{ (or) } {}^{\circ}C/W.}$$

Electrical	Thermal
ΔV	ΔT
I	Q
R_{elect}	$R_{thermal}$

Conditions Apply for Thermal Resistance :–

1) steady state
2) 1-D → No 3rd point as in previous Assumptions
3) No q_g
4) 'K' is const.

plane slab with convection at Boundaries :-



→ According initial & Boundary concept with Energy Balance concepts

1) → $Q_{conv} = h_a A (T_a - T_1)$

$$T_a - T_1 = Q\left[\frac{1}{h_a A}\right] \Rightarrow \Delta T = Q(R_{th})$$

$\therefore R_{th_{conv}} = \frac{1}{h_a A}$.

2) → $Q_{cond} = \frac{KA(T_1 - T_2)}{L}$

$$T_1 - T_2 = Q\left[\frac{L}{KA}\right] \Rightarrow \Delta T = Q(R_{th})$$

$\therefore R_{th_{cond}} = \frac{L}{KA}$.

3) → $Q_{conv} = h_b A (T_2 - T_b)$

$$T_2 - T_b = Q\left[\frac{1}{h_b A}\right] \Rightarrow \Delta T = Q(R_{th})$$

$\therefore R_{th_{conv}} = \frac{1}{h_b A}$.

$\therefore Q_{conv} + Q_{cond} + Q_{conv}$

$$\Rightarrow T_a - \cancel{T_1} + \cancel{T_1} - \cancel{T_2} + \cancel{T_2} - T_b = Q\left[\frac{1}{h_a A} + \frac{L}{KA} + \frac{1}{h_b A}\right]$$

$$\Rightarrow \quad T_a - T_b = Q\left[\frac{1}{h_a A} + \frac{L}{kA} + \frac{1}{h_b A}\right]$$

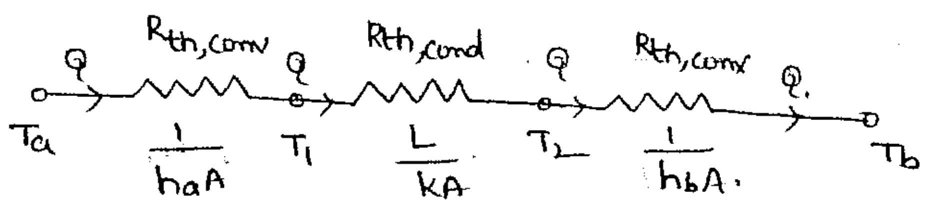
$$\boxed{\Delta T_{overall} = Q\left[R_{effective}\right]}$$

where

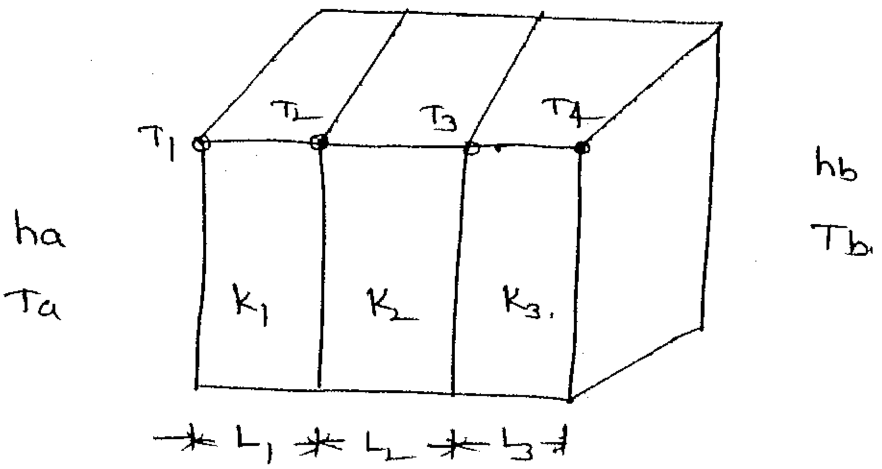
$$\frac{L}{kA} = R_{th\,cond} = \text{Conductive Resistance}$$

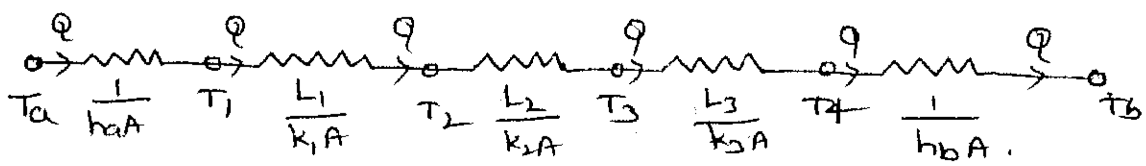
$$\frac{1}{hA} = R_{th,conv} = \text{Convective Resistance}$$

Circuit diagram :-



Composit slab :-





$$\therefore \quad \boxed{T_a - T_b = Q\underbrace{\left[\frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{1}{h_b A}\right]}_{R_{eff.}}}$$

↳ Effective Thermal Resistance

Overal Heat transfer Co-efficient :- [U]

$$Q = UA(T_a - T_b) \qquad [\because \text{like } Q = hA(T_a - T_b)]$$

$$T_a - T_b = \frac{Q}{UA}$$

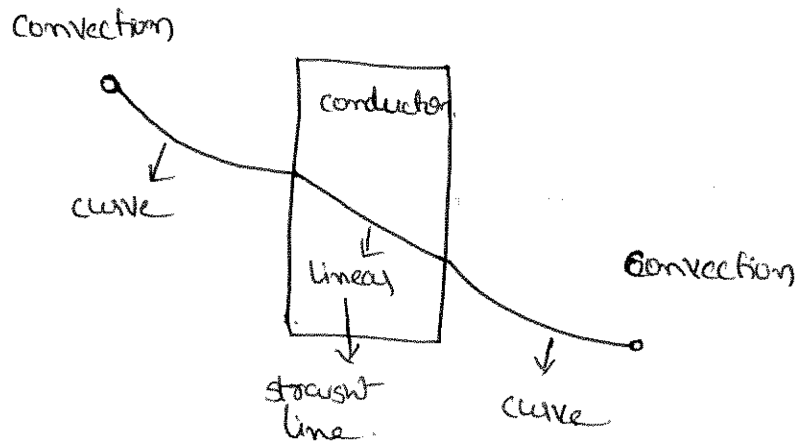
$$A_1 = A_2 = A_3 = A.$$

$$\frac{\cancel{Q}}{U\cancel{A}} = \cancel{Q}\left[\frac{1}{h_a \cancel{A}} + \frac{L_1}{k_1 \cancel{A}} + \frac{L_2}{k_2 \cancel{A}} + \frac{L_3}{k_3 \cancel{A}} + \frac{1}{h_b \cancel{A}}\right]$$

$$\boxed{\frac{1}{U} = \frac{1}{h_a} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_b}}$$

Note :- The contact B/W The composit ~~wall~~ slab surfaces is assumed to be perfect and temp profile is continues [other wise it is variable at surface gap is there and it is disscussed in another topic ie Thermal contact Resistance]

Temparature profile with Convection at Boundaries :-



Conventional questions :-

1) Determine the steady state heat transfer through a double window 0.8m height and 1.5m width, consisting of two 4mm thick glasses layers ($k = 0.78\ w/m^\circ c$) seperated by a 10mm thick stagnent layers of air ($k = 0.026\ w/m^\circ c$) inside temp of room air is maintained at $20^\circ c$ with convective heat transfer co-efficient of $10 w/m^2 {}^\circ c$, outside air temp is $-10^\circ c$ with convective heat transfer co-effcient of $40 w/m^2 {}^\circ c$ also. Determine the overal heat transfer co-efficient.

solve the problems for HT if a single window of 8mm thickness is used instead of double window and make suitable conclusions

Data :-

$H = 0.8m$, $B = 1.5m$

$\therefore A = 0.8 \times 1.5\ m^2$

$A = 1.2\ m^2$

$L_1 = L_3 = 4\ mm$
$= 4 \times 10^{-3} m$

$k_1 = k_3 = 0.78\ w/m^\circ c$.

$L_2 = 10mm$
$= 10 \times 10^{-3} m$

$k_2 = 0.026\ w/m^\circ c$.

$Ta = 20^\circ c$, $ha = 10 w/m^2 {}^\circ c$

$Tb = -10^\circ c$, $hb = 40 w/m^2 {}^\circ c$.

Find :-

1) case 1)

a) $Q = ?$

b) $U = ?$

case 2)

a) $Q = ?$

b)

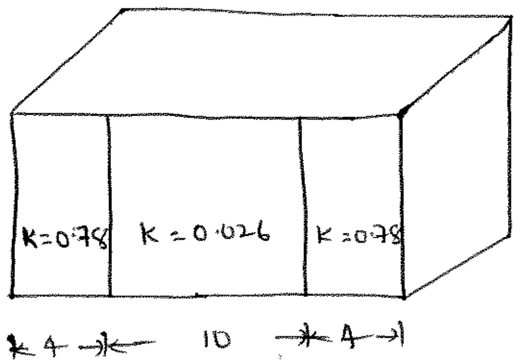
Schematic :-

case 1

Room

$T_a = 20^\circ C$

$h_a = 10$



outside air

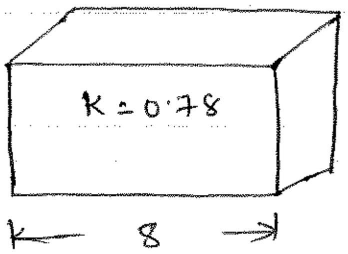
$T_b = -10^\circ C$

$h_b = 40^\circ C.$

case 2

$T_a = 20$

$h_a = 10$



$T_b = -10$

$h_b = 40.$

Assumptions :-

1) steady state

2) One Dimension Heat conduction.

3) constant properties

4) Homogenious Material

5) NO Heat generation

6) contact Resistance is negligible.

Sol:—

Case1:— $\Delta T = Q\, R_{effective}$

$$T_a - T_b = Q \left[\frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{1}{h_b A} \right]$$

$$20 - (-10) = Q \left[\frac{1}{10 \times 1.2} + \frac{4 \times 10^{-3}}{0.78 \times 1.2} + \frac{10 \times 10^{-3}}{0.026 \times 1.2} + \frac{4 \times 10^{-3}}{0.78 \times 1.2} + \frac{1}{40 \times 1.2} \right]$$

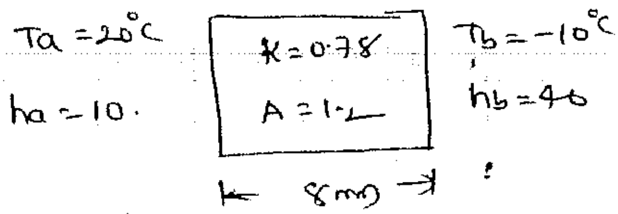
$$\therefore Q = 69.2\ W.$$

$$Q = UA(T_a - T_b)$$

$$69.2 = U \times 1.2\,(20 - (-10))$$

$$U = 1.92\ W/m^2K.$$

case 2:—



$$\Delta T = Q^1 R_{eff}$$

$$T_a - T_b = Q^1 \left[\frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{1}{h_b A} \right]$$

$$20 - (-10) = Q^1 \left[\frac{1}{10 \times 1.2} + \frac{8 \times 10^{-3}}{0.78 \times 1.2} + \frac{1}{40 \times 1.2} \right]$$

$$\therefore Q^1 = 266.1\,W.$$

Conclusions:—

Generally double windows are prefered, because they not only reduce heat transfer, they also reduce noise. This is because b/w two layers there is air which offers high thermal resistance due to it's lower thermal conductivity. Similarly in winter double blankets are prefered instead of single layer blanket.

2) A Composit wall consists of 10cm layer of brick ($k = 0.7\,W/mK$) and 3cm thick plaster ($k = 0.5\,W/mK$). An Insulating Material of $k = 0.08\,W/mK$ is to be added to reduce heat transfer through the wall by 70%. Determine the thickness of insulating layer.

Data :-

$L_1 = 10cm = 10^{-2}m$ $\quad$ $L_2 = \cancel{0.5}\, 3cm = 3\times10^{-2}m$

$k_1 = 0.7\,W/mk$ $\quad$ $k_2 = 0.5\,W/mk$.

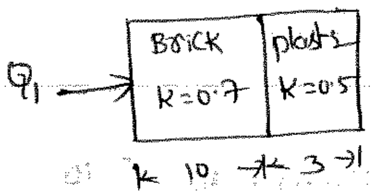
If insulating Material is added (k_3) = $0.08\,W/mk$.

Find :-

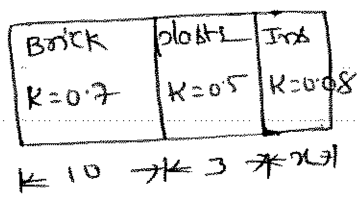
1) Determine the thickness of insulating Material if 70% heat is reduced.

schematic :-

case 1 :-



case 2 :-



Assumptions :-

1) steady state
2) 1-D
3) const properties
4) Homogenious Material
5) No 'q_g'
6) neglect contact Resistance

Sol:-

case 1:-

$$\Delta T = Q_1 R$$

$$\Delta T = Q_1 \left[\frac{L_1}{K_1 A} + \frac{L_2}{K_2 A}\right]$$

$$\Delta T = Q_1 \left[\frac{0.1}{0.7\times 1} + \frac{0.03}{0.5\times 1}\right]$$

$\cancel{Q_1 = 4.92/\Delta T}$ $\quad \Delta T = Q_1 [0.202]$

case 2:-

$$\Delta T = Q_2 \left[\frac{0.1}{0.7\times 1} + \frac{0.03}{0.5\times 1} + \frac{x}{0.08\times 1}\right].$$

Q_2 = 70% Lost of heat ~~is~~ with 1st case

$$Q_2 = 0.3 Q_1$$

$$\Delta T = 0.3 Q_1 \left[0.202 + \frac{x}{0.08}\right]$$

$$\cancel{Q_1} \times 0.202 = 0.3 \cancel{Q_1} \left[0.202 + \frac{x}{0.08}\right]$$

$$x = 3.79\,cm$$

3) A very thin square plate heater ~~of~~ (15cm) 10 ~~of~~ 10 ~~1kW is~~ inserted b/w two slabs, slab A is 2cm thick ($k = 50\,W/mK$) and slab B is 1cm thick ($k = 0.2\,W/mK$). The outside heat transfer coefficient on A & B are 200 & 50 W/m^2K respectively. Temp of the surrounding air is 25°C. If the rating of the heater is 1kW

Find

1) Max. temp in the system.
2) outside surface temp of two slabs
3) Draw equivalent circuit diagram for the system

Data

Area $(A) = 15\,cm^2 = 225 \times 10^{-4}\,m^2$

$L_A = 2\,cm$, $L_B = 1\,cm$

$= \cancel{0.0\,m}\ 0.02\,m$ $= \cancel{0.1\,m}\ 0.01\,m$

$K_A = 50\,W/mK$ $K_B = 0.2\,W/m.K.$

$h_a = 200\,W/m^2K$ $T_a = 25°C.$

$h_b = 50\,W/m^2K$ $Q_0 = 1\,kW$

$= 1000\,W.$

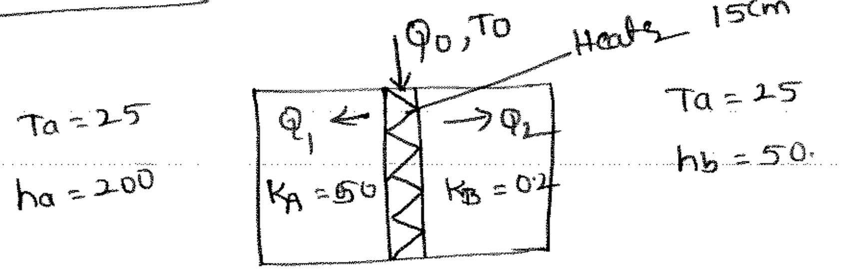
Find :-

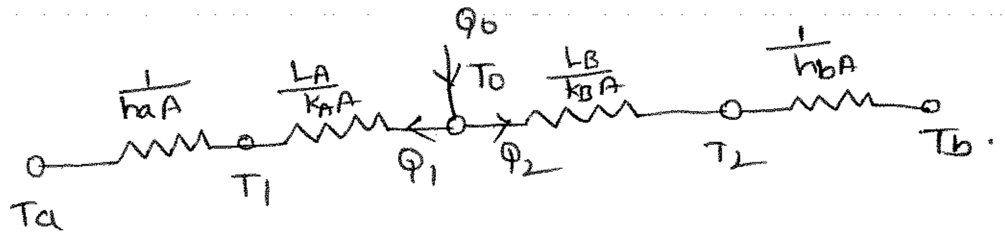
1) $T_{max} = ?$

2) outside temp's $(T_1 \,\&\, T_2) = ?$

3) circuit Diagram

Schematic :-





Assumptions :-

Sol:-

$$Q = Q_1 + Q_2$$

$$1000 = Q_1 + Q_2 \quad \text{--- (1)}$$

$$\Delta T = QR$$

$$T_0 - T_a = Q_1\left[\frac{L_A}{k_A A} + \frac{1}{h_a A}\right]$$

$$T_0 - 25 = Q_1\left[\frac{0.02}{50\times225\times10^{-4}} + \frac{1}{200\times225\times10^{-4}}\right]$$

$$T_0 - 25 = 0.24\,Q_1 \quad \text{--- (2)}$$

$$T_0 - T_b = Q_2\left[\frac{L_B}{k_B A} + \frac{1}{h_b A}\right]$$

$$T_0 - 25 = Q_2\left[\frac{0.01}{0.2\times225\times10^{-4}} + \frac{1}{50\times225\times10^{-4}}\right]$$

$$T_0 - 25 = 3.11\,Q_2 \quad \text{--- (3)}$$

From equations (2) & (3)

$$0.24\,Q_1 = 3.11\,Q_2$$

$$Q_1 = \frac{3.11}{0.24}\,Q_2$$

$$Q_1 = 12.95\,Q_2$$

$$\therefore Q_1 + Q_2 = 1000$$

$$12.95\,Q_2 + Q_2 = 1000$$

$$\therefore Q_2 = 71.64\,W.$$

$$\therefore Q_1 = 927.7\,W.$$

$$T_0 - 25 = 3.11\, Q_2$$

$$T_0 = 247.8^\circ C$$

$$\therefore T_{max} = T_0 = T_{Heater} = \underline{\underline{247.8^\circ C}}$$

$$T_0 - T_1 = Q_1 \left[\frac{L_A}{K_A A} \right]$$

$$247.8 - T_1 = 927.7 \left[\frac{0.02}{50 \times 225 \times 10^{-4}} \right]$$

$$\underline{\underline{T_1 = 231.3^\circ C}}$$

^{11}ly

$$T_0 - T_2 = Q_2 \left[\frac{L_B}{K_B A} \right]$$

$$247.8 - T_2 = 71.64 \left[\frac{0.01}{0.2 \times 225 \times 10^{-4}} \right]$$

$$\underline{\underline{T_2 = 88.6^\circ C}}$$

4) The Inside temp of a furnace wall 200m thick is 1350°C. The Thermal conductivity of wall is 1.35 W/mK. The heat transfer co-efficient of outside surface is a function of temp difference is given by $h = 7.85 + 0.084\Delta T$ where ΔT is the temp difference is outside wall surface and surroundings. Determine the rate of heat transfer per unit area, if surrounding temp is 40°C.

Data :-

$T_1 = 1350^\circ C$.

$L = 200mm = 0.2m$

$k = 1.35\ W/mK$.

h_a ~~b/out~~ = ~~7.85~~ $7.85 + 0.08\,\Delta T$

ΔT = Temp difference b/w. wall & outer surface

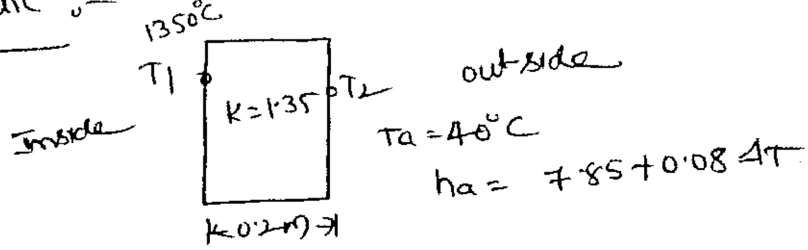
$= T_2 - T_a$

$T_a = 40^\circ C$.

Find :-

1) $Q = ?$

schematic :-



Assumptions :-

1)

2)

3)

4)

Sol:-

$$Q_{cond} = Q_{conv}$$

$$\cancel{\Delta T} = \cancel{QR}. \quad \frac{KA\,(T_1-T_2)}{L} = h_a\,A\,(T_1-T_2)$$

$$\frac{1.35\times 1\,(1350-T_2)}{0.2} = 7.85+0.08\,\Delta T\,A\,(T_2-T_a)$$

$$\frac{1.35\times 1\,(1350-T_2)}{0.2} = 7.85+0.08\,(T_2-40)\times 1\,(T_2-40)$$

$$\therefore \quad T_2 = 293.5^\circ C$$

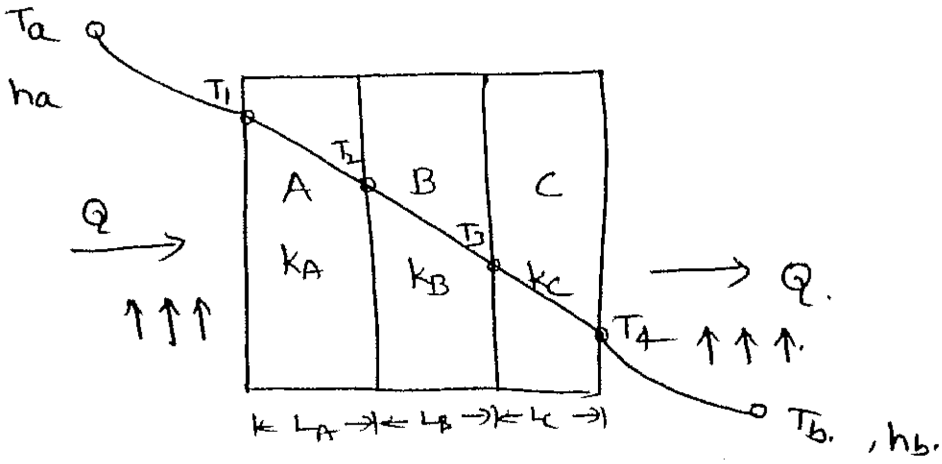
$$\therefore \quad \Delta T = QR$$

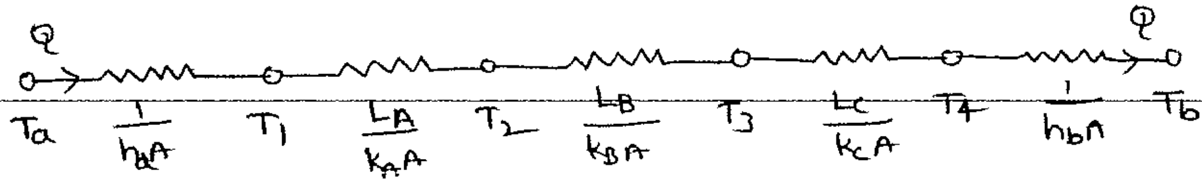
$$Q = \frac{\Delta T}{R}. \qquad R = \frac{L}{KA}$$

$$Q = \frac{1350-293.5}{\frac{0.2}{1.35\times 1}}$$

$$Q = 7131.3 \ \omega/m^2$$

plane slabs in series :-





$$(\Delta T)_{overal} = Q \,\Sigma R_{th}$$

$$Q = \frac{(\Delta T)_{overal}}{\Sigma R_{th}}$$

$$(\Delta T)_{overal} = T_a - T_b$$

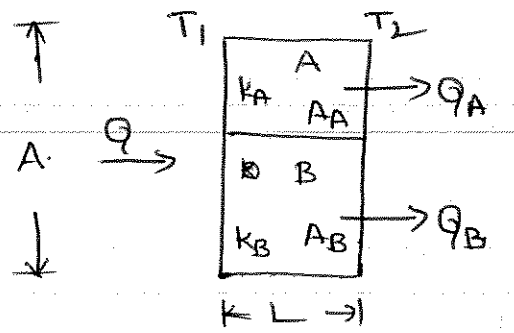
$$\Sigma R_{th} = \frac{1}{h_a A} + \frac{L_A}{K_A A} + \frac{L_B}{K_B A} + \frac{L_C}{K_C A} + \frac{1}{h_b A}.$$

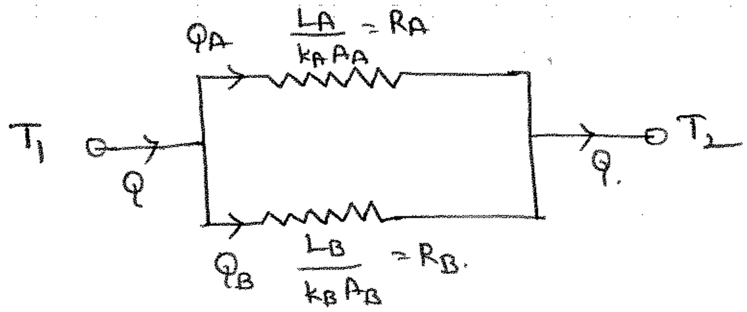
$$\therefore Q = \frac{T_a - T_b}{\frac{1}{h_a A} + \frac{L_A}{K_A A} + \frac{L_B}{K_B A} + \frac{L_C}{K_C A} + \frac{1}{h_b A}}$$

$\therefore$ Heat transfer associated with each layer is

$$Q_1 = \frac{T_a - T_1}{\frac{1}{h_a A}} = \frac{T_1 - T_2}{\frac{L_A}{K_A A}} = \frac{T_2 - T_3}{\frac{L_B}{K_B A}} = \frac{T_3 - T_4}{\frac{L_C}{K_C A}} = \frac{T_4 - T_b}{\frac{1}{h_b A}}.$$

Heat conduction Through parallel slabs :-





→ Since the heat is conducted through two different paths b/w the same temp difference.

∴ The total rate of HT is sum of heat flow through areas A_A & A_B

$$\Delta T = QR \Rightarrow Q = \frac{\Delta T}{R}$$

$$Q = Q_A + Q_B$$

$$= \frac{T_1 - T_2}{\frac{L_A}{k_A A_A}} + \frac{T_1 - T_2}{\frac{L_B}{k_B A_B}}$$

$$= T_1 - T_2 \left[\frac{1}{\frac{L_A}{k_A A_A}} + \frac{1}{\frac{L_B}{k_B A_B}} \right]$$

$$\because R_A = \frac{L_A}{k_A A_A}, \quad R_B = \frac{L_B}{k_B A_B}$$

$$\cancel{\frac{dT}{dx} = -1236\ K/m}$$

$$Q = (T_1 - T_2)\left[\frac{1}{R_A} + \frac{1}{R_B}\right]$$

$$\boxed{Q = (T_1 - T_2)\left[\frac{1}{R_{eq}}\right]}$$

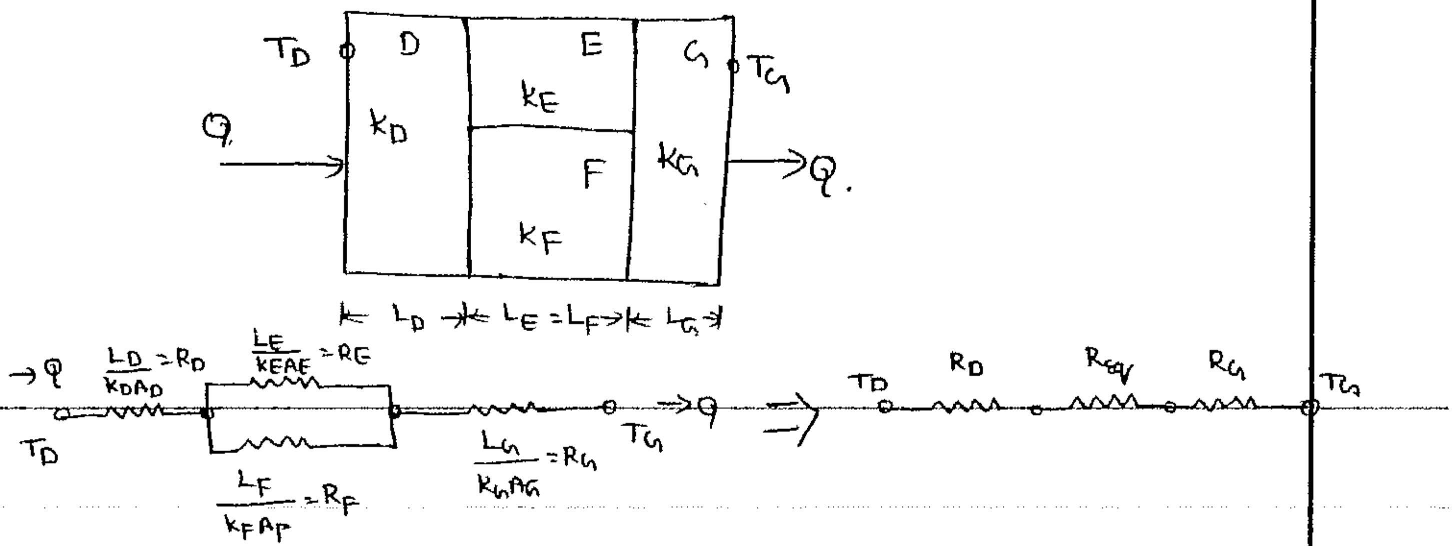
$$\boxed{\therefore \frac{1}{R_{eq}} = \frac{1}{R_A} + \frac{1}{R_B}.}$$

$$R_{eq} = \frac{1}{\frac{1}{R_A} + \frac{1}{R_B}}$$

$$\boxed{R_{eq} = \frac{R_A R_B}{R_A + R_B}}$$

Composit wall in series and parallel :-

Consider a composit wall with series & parallel configuration as show in fig below



Let

A_E = HT area of Layer $\underline{E}$

A_F = HT area of Layer $\underline{F}$

$A = A_E + A_F$

The equivalent resistance can be calculated of parallel resistances in Fig.

$$\frac{1}{R_{eq}} = \frac{1}{R_E} + \frac{1}{R_F}$$

$$\frac{1}{R_{eq}} = \frac{K_E A_E}{L_E} + \frac{K_F A_F}{L_F}$$

$$\boxed{R_{eq} = \frac{R_E R_F}{R_E + R_F} = \frac{1}{\frac{K_E A_E}{L_E} + \frac{K_F A_F}{L_F}}}$$

$$\therefore R_{total} = \Sigma R_{th} = R_D + R_{eq} + R_G$$

$$\boxed{R_{total} = \Sigma R_{th} = \frac{L_D}{K_D A_D} + \frac{1}{\frac{K_E A_E}{L_E} + \frac{K_F A_F}{L_F}} + \frac{L_G}{K_G A_G}}$$

Conventional Questions :-

1) The Large furnace wall consists of 250mm thick common brick layer ($k = 0.65$ W/m·K), lined on inside with 300mm thick layer of Magnesite bricks ($k = 11.5$ W/mK). The Inner side of the furnace is exposed to hot gases at 1400°C with convective heat transfer co-efficient of

17.5 W/m²K, and radiative heat transfer co-efficient of 23.2 W/m²K. The temperature of the surrounding air is 30°C with convective heat transfer co-efficient of 7.5 W/m²K and radiative heat transfer co-efficient of 11.5 W/m²K.

calculate

1) Rate of HT through the wall per unit area

2) Max temp to which the common brick is subjected

Data :-

$L_1 = 300mm = 0.3m$ $\quad$ $L_2 = 250mm = 0.25m$

$K_1 = 11.5\ W/mK$ $\quad$ $k_2 = 0.65\ W/mK.$

$T_a = 1400°C.$ $\quad$ $T_b = 30°C$

$h_{c_1} = 17.5 W/m^2K$ $\quad$ $h_{c_2} = 7.5 W/m^2K$

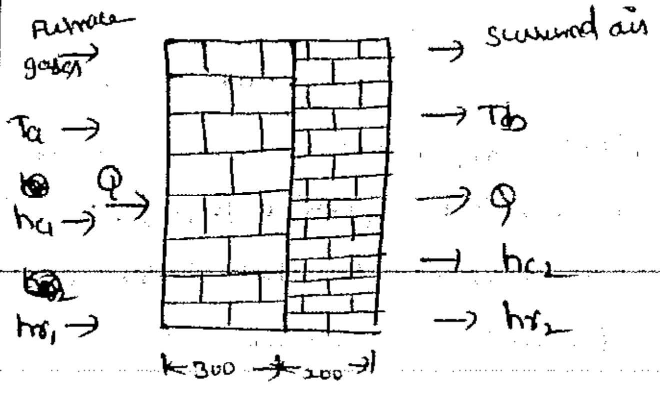
$h_{r_1} = 23.2\ W/m^2K.$ $\quad$ $h_{r_2} = 11.5\ W/m^2K.$

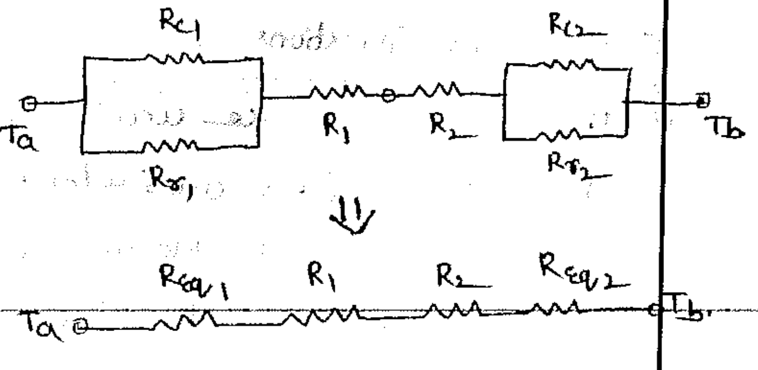
Find :-

1) $\frac{Q}{A} = q = ?$

2) Tmax at common bricks $[T_{Comm\ Brick}]$

Schematic :-





Sol :-

1) $\Delta T = Q R_{total}$

$$Q = \frac{\Delta T}{R_{total}}$$

$\therefore$

$$q = \frac{\Delta T}{\Sigma R_{th}/m^2 \text{ area}} \quad \text{~~per m}^2\text{~~ area}$$

R_{c_1} & R_{r_1}
R_{c_2} & R_{r_2} } → In parallel.

$$\Sigma R_{th} = R_{eq_1} + R_1 + R_2 + R_{eq_2}.$$

All thermal Resistances are per unit area

$$\frac{1}{R_{eq_1}} = \frac{1}{R_{c_1}} + \frac{1}{R_{r_1}}$$

$$\frac{1}{R_{eq_2}} = \frac{1}{R_{c_2}} + \frac{1}{R_{r_2}}$$

$$\therefore R_{c_1} = \frac{1}{h_{c_1}} \Rightarrow R_{c_1} = \frac{1}{17.5} \Rightarrow R_{c_1} = 0.0571 \ m^2 K/W.$$

$$R_{r_1} = \frac{1}{h_{r_1}} \Rightarrow R_{r_1} = \frac{1}{23.2} \Rightarrow R_{r_1} = 0.0431 \ m^2 K/W$$

$$R_1 = \frac{L_1}{k_1} \Rightarrow R_1 = \frac{0.3}{11.5} \Rightarrow R_1 = 0.026 \ m^2 K/W.$$

$$R_2 = \frac{L_2}{k_2} \Rightarrow R_2 = \frac{0.25}{0.65} \Rightarrow R_2 = 0.3846 \ m^2 K/W.$$

$$R_{c_2} = \frac{1}{h_{c_2}} \Rightarrow R_{c_2} = \frac{1}{7.5} \Rightarrow R_{c_2} = 0.1333 \ m^2 K/W$$

$$R_{r_2} = \frac{1}{h_{r_2}} \Rightarrow R_{r_2} = \frac{1}{11.5} \Rightarrow R_{r_2} = 0.087 \ m^2 K/W.$$

$$\therefore \frac{1}{R_{eq_1}} = \frac{1}{0.0571} + \frac{1}{0.0431} \Rightarrow \frac{1}{R_{eq_1}} = 40.7$$

$$\Rightarrow R_{eq_1} = 0.0245 \ m^2 K/W$$

$$R_{eq2} = \frac{1}{R_{c2}} + \frac{1}{R_{r2}} \Rightarrow R_{eq2} = \frac{1}{0.1300} + \frac{1}{0.087}$$

$$\therefore R_{eq} = 0.05263 \ m^2 \ K/W.$$

$$\therefore R_{total} = \Sigma R_{th} = 0.0245 + 0.026 + 0.3846 + 0.05263$$

$$\Sigma R_{th} = 0.4877 \ m^2 \ K/W.$$

$$\therefore q = \frac{T_a - T_b}{\Sigma R_{th}} \Rightarrow q = \frac{1400 - 300}{0.4877}$$

$$\therefore q = \underline{\underline{2808.92 \ W/m^2}}$$

2) The Max Temp is to be at Comon Brick ie,.

$$q = \frac{T_a - T_1}{R_{eq1} + R_1}$$

$$2808.92 = \frac{1400 - T_1}{0.0245 + 0.026.}$$

$$\therefore T_1 = \underline{\underline{1258.15^\circ C}}$$

2) Conside a 5m height and 8m long and 0.22m thick wall whose representation is shown in fig. The thermal conductivity of various materials used are $k_A = k_F = 2$, $k_B = 8$, $k_C = 20$, $k_D = 15$, & $k_E = 35$ W/mK. The left surface is of the wall is maintained at a uniform temp of 300°C. The right surface is exposed to convection environment

at 50°C with h = 20W/m²K. Determine

a) one Dimensional Heat transfer rate through the wall

b) Temp at the point where section B, D and E meet, and

c) temp drop accross the section F.

Data :-

$$A_{wall} = 5m \times 8m \rightarrow \text{Total Area}$$
$$= 40m^2.$$

$$L_{total} = 0.22m.$$

$k_A = k_F = 2 W/mK$, $k_B = 8 W/mK$, $k_C = 20 W/mK$

$k_D = 15 W/mK$, $k_E = 35 W/mK$.

$T_1 = 300°C$, $T_b = 50°C$
$h_b = 20 W/m^2K$.

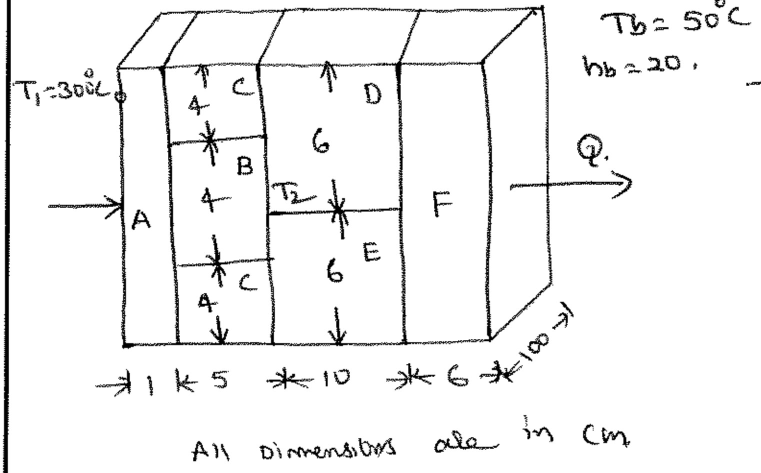
Find :-

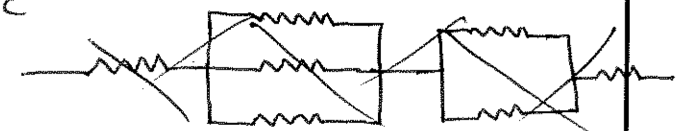
1) $Q = ?$

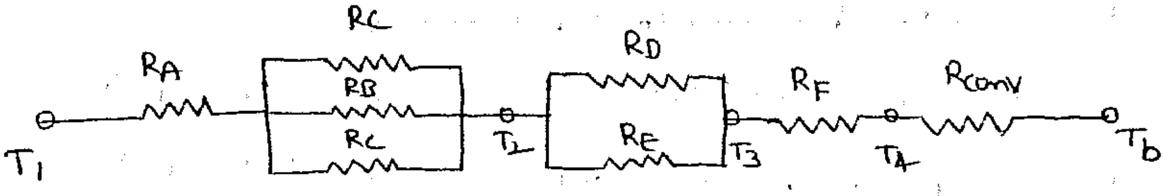
2) Temp at the section B, D & E Meet.

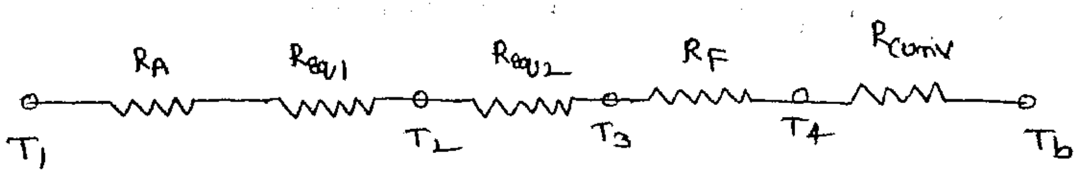
3) Temp drop at the section F.

Schematic :-









Assumptions :-

1) steady state condition
2) One Dimensional Heat conduction.
3) No q_g
4) constant properties
5) Homogenious material
6) No contact Resistance

Sol :-

1) $\Delta T = Q_{\cancel{effect}} R_{eff.}$

$$Q = \frac{\Delta T}{R_{effect}}$$

$\cancel{Q=}$ $\Delta T = T_1 - T_a$

$R_{effective} = \Sigma R_{th} = R_{total} = R_A + R_{eq_1} + R_{eq_2} + R_F + R_{conv}$

$R_A = \frac{L_A}{K_A A} \Rightarrow \cancel{R_A} = \cancel{\frac{0.01}{2 \times 40}} \Rightarrow \cancel{R_A =}$

$R_B =$

The cross-sectional areas of Representative sections are to be

$A_1 = \cancel{0.1m \times 0.12m = 0.12m^2}$

Cross section Area of portions (A) & (F) $= 1 \times 0.12$
$= 0.12 m^2$

Cross section Area of portions B & C $= 1 \times 0.04$
$= 0.04 m^2$

Cross section Area of portions D & E $= 1 \times 0.06$
$= 0.06 m^2$

$$\therefore R_A = \frac{L_A}{K_A A_A} \Rightarrow R_A = \frac{0.01}{2 \times 0.12} \Rightarrow R_A = 0.04167 \ K/W.$$

$$R_B = \frac{L_B}{K_B A_B} \Rightarrow R_B = \frac{0.05}{8 \times 0.04} \Rightarrow R_B = 0.15625 \ K/W.$$

$$R_C = \frac{L_C}{K_C A_C} \Rightarrow R_C = \frac{0.05}{20 \times 0.04} \Rightarrow R_C = 0.0625 \ K/W$$

$$R_D = \frac{L_D}{K_D A_D} \Rightarrow R_D = \frac{0.1}{15 \times 0.06} \Rightarrow R_D = 0.111 \ K/W.$$

$$R_E = \frac{L_E}{K_E A_E} \Rightarrow R_E = \frac{0.1}{35 \times 0.06} \Rightarrow R_E = 0.0476 \ K/W.$$

$$R_F = \frac{L_F}{K_F A_F} \Rightarrow R_F = \frac{0.06}{20 \times 0.12} \Rightarrow R_F = 0.\cancel{4167}\,0.25 \ K/W.$$

$$R_{conv} = \frac{1}{h_a A_a} \Rightarrow R_{conv} = \frac{1}{20 \times 0.12} \Rightarrow R_{conv} = 0.4167 \ K/W$$

Here R_B & R_C's are parallel, & R_D & R_E are parallel

$$\therefore \frac{1}{R_{eq1}} = \frac{1}{R_C} + \frac{1}{R_B} + \frac{1}{R_C}$$

$$= \frac{1}{0.0625} + \frac{1}{0.15625} + \frac{1}{0.0625}$$

$$\frac{1}{R_{eq1}} = \cancel{30.017}\ 38.4$$

$$\Rightarrow R_{eq1} = 0.026 \ K/W.$$

$$R_{EQ2} = \frac{1}{R_D} + \frac{1}{R_E}$$

$$= \frac{1}{0.111} + \frac{1}{0.0476}$$

$$R_{EQ2} = \frac{1}{30.017} \Rightarrow R_{EQ2} = 0.0333 \ K/W.$$

$$\therefore \Sigma R_{th} = 0.04167 + 0.026 + 0.0330 + 0.25 + 0.4167$$

$$= 0.7677 \ K/W.$$

$\therefore$ Heat transfer in Representive section of wall

$$\therefore Q = \frac{T_1 - T_a}{\Sigma R_{th}}$$

$$Q = \frac{300-50}{0.7677} \Rightarrow Q = \underline{\underline{325.65 \ W}}$$

$\therefore$ Heat transfer through total surface of the wall

$$= Q \times \frac{\text{wall area (total)}}{\text{Representive area}}$$

$$= 325.65 \times \frac{40}{0.12}$$

$$= \underline{\underline{108.55 \ KW}}$$

2) Temp at R_A & R_{EQ1}

$$Q = \frac{T_1 - T_2}{R_A + R_{EQ1}} \Rightarrow \overset{325.65}{\cancel{Q\ 108.55}} = \frac{300 - T_2}{0.04167 + 0.026} \Rightarrow$$

$$\therefore T_2 = \underline{\underline{278^\circ C}}$$

3) Temp drop accross section $\underline{F}$

$$Q = \frac{T_3 - T_4}{R_F} \Rightarrow Q = \frac{\Delta T_F}{R_F} \Rightarrow \Delta T_F = Q \times R_F$$

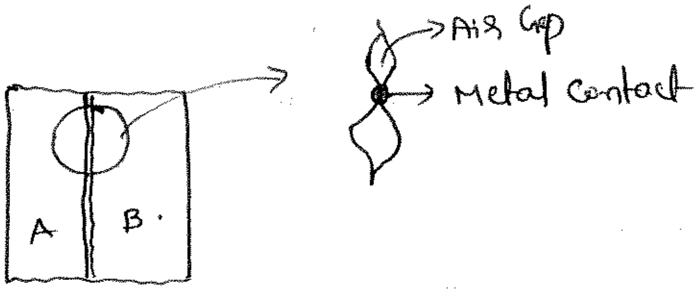
$$\therefore \Delta T_F = 325.65 \times 0.25 \Rightarrow \underline{\underline{\Delta T_F = 81.41^\circ C}}$$

Thermal contact Resistance :-

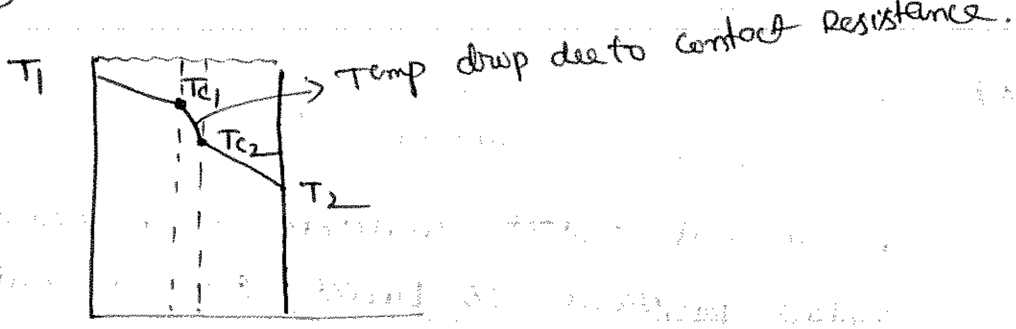
→ In the Analysis of heat conduction through multi ~~cylinders~~ layers solids, we assumed perfect contact at the interface of two layers, and thus no temparature drop at the interface.

→ when two surfaces are brought in contact, so the contact may not be perfect due to with numerous peaks ~~or~~, valleys ~~and~~, voids of the surface is roughness and hence the gap is filled with air (gas) ~~which offers~~ it acts as insulation because of Low thermal conductivity

→ This opposing thermal conductivity due to Imperfect contact is known as Thermal contact Resistance (R_c)



→ Because of this, the temp profile is not continuously and there is a significant temp drop at the ~~Contin~~ contacting surfaces



→ If the heat flux through two solid surfaces in contact is 'q' and the temp diffrence across the contact (fluid gap) is ΔT ($T_{c_1} - T_{c_2}$) is as shwo in fig.

∴ The contact Resistance

$$\boxed{R_{contact} = \frac{T_{c_1} - T_{c_2}}{q}} - \frac{K}{W/m^2} \Rightarrow \frac{m^2 K}{W}.$$

Like wise $R_{th} = \frac{\Delta T}{q}$ But per unit Area

$$R_c = \frac{\text{Temp drop across contact surfaces}}{\text{Heat Flux.}}$$

→ The value of thermal contact Resistance depends on the surface roughness and the material properties as well as temperature and pressure at the interface and the type of fluid trapped at the interface.

→ The Thermal contact Resistance can be Minimized by.

1) By ~~Ag~~ Increase in contact pressure can reduce the contact Resistance Drastically

2) By Applying a thermally conducting liquid called thermal ~~gases~~ Fluids such as silicon oil on the surfaces before they are pressed against each other

3) By Replacing the air at the interface by a better thermal conducting gas such as Helium or hydrogen

4) By Inserting a soft metallic foil such as tin, silver, copper, nickle or Aluminium b/w the two surfaces

Note :- The Thermal contact conductance is highest (and thus contact Resistance is Lowest) for soft materials with smooth surfaces at high pressure.

conventional questions :-

1) consider a plane ~~of~~ composit wall that is composssed of two materials of thermal conductivities $K_A = 0.1 W/mK$ and $K_B = 0.04 W/mK$ and thickness $L_A = 10mm$ & $L_B = 20mm$., The Thermal contact Resistance at the interface b/wo two materal is $0.06°C/W$. Materal A is surrounded by a fluid at 200°C for which $h = 10W/m^2K$ materal B is surrounded by a fluid at 40°C for which $h = 21W/m^2K$. what is the rate of heat transfer through wall is 2m height and 2.5m wide also calculate overal heat transfer co-efficient and sketch the temp distribution

Data

$K_A = 0.1 W/mK$ $\qquad$ $K_B = 0.04 W/mK$

$L_A = 10mm$ $\qquad$ $L_B = 20mm$

$= 10 \times 10^{-3} m$ $\qquad$ $= 20 \times 10^{-3} m$

$R_c = 0.06°C$.

$T_a = 200°C$ $\qquad$ $T_b = 40°C$

$h_a = 10W/m^2K$ $\qquad$ $h_b = 21W/m^2K$.

$A = 2 \times 2.5 = 5m^2$

Find:-

1) $Q = ?$

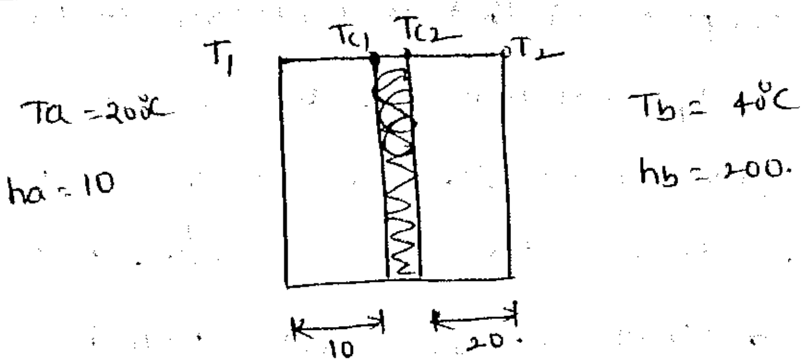
2) $U = ?$

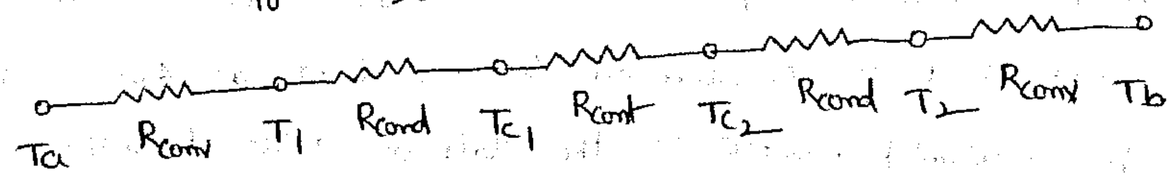
3) sketch the temp distribution [Find All temp's]

Assumptions :-

1) steady state
2) 1-D
3) No q_g
4) ~~No~~ const properties
5)

Schematic :-





Sol :-

1) $\Delta T = Q R_{eff}$

$$Ta - Tb = Q\left[\frac{1}{h_a A} + \frac{L_1}{K_1 A} + R_C + \frac{L_2}{k_2 A} + \frac{1}{h_b A}\right]$$

$$200 - 40 = Q\left[\frac{1}{10\times5} + \frac{10\times10^{-3}}{0.1\times5} + 0.06 + \frac{20\times10^{-3}}{0.04\times5} + \frac{1}{20\times5}\right]$$

$$Q = \underline{761.9 W}$$

2)

$$Q = UA\Delta T$$

$$761.9 = U\times5\,(200-40)$$

$$U = 0.952\ W/m^2K$$

3) For temp distribution we should find All temperatures

$$Q = hA\Delta T$$

$$761.9 = 10\times5\,(200 - T_1)$$

$$T_1 = \underline{184.7°C}$$

'Q' Every where is same, because steady state

(or) $\Delta = QR$

$$200 - T_1 = 761.9\left[\frac{1}{10\times5}\right]$$

$$\Delta T = QR \Rightarrow T_1 - T_{c_1} = Q\left[\frac{L}{KA}\right]$$

$$184.7 - T_{c_1} = 761.9\left[\frac{10\times10^{-3}}{0.1\times5}\right]$$

$$\underline{T_{c_1} = 169.5^\circ C}$$

$$\Delta T = QR$$

$$T_{c_1} - T_{c_2} = Q \times R_{contact}$$

$$169.5 - T_{c_2} = \cancel{169}\ 761.9 \times 0.06$$

$$\underline{T_{c_2} = 123.7^\circ C}$$

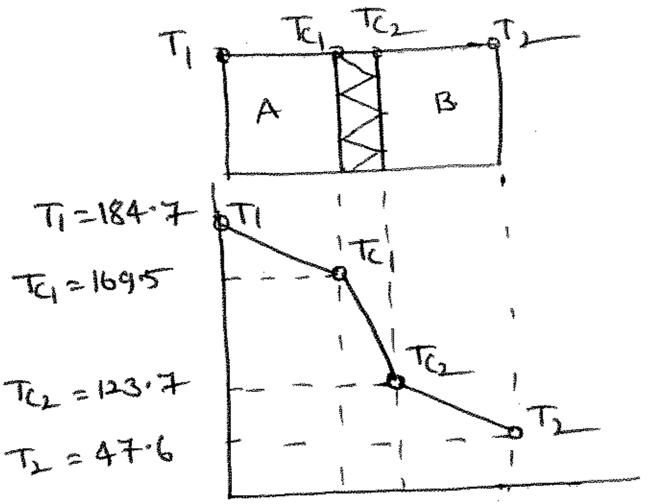
$$\Delta T = QR$$

$$T_{c_2} - T_2 = Q\left[\frac{L}{KA}\right]$$

$$123.7 - T_2 = 761.9\left[\frac{20\times10^{-3}}{0.04\times5}\right]$$

$$\underline{T_2 = 47.6^\circ C}$$

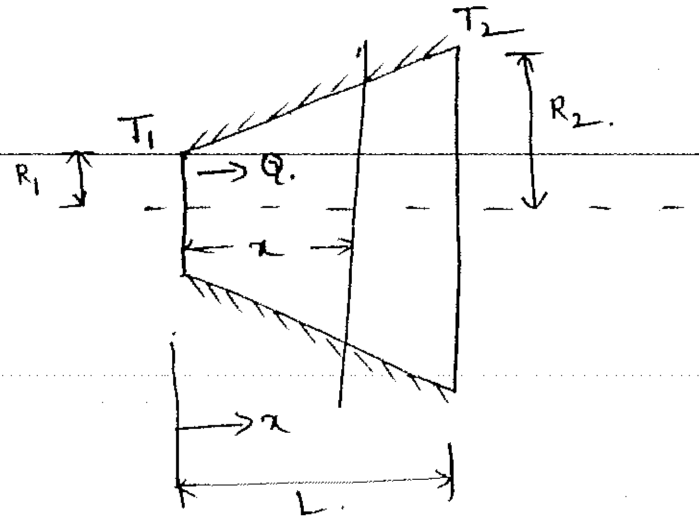
Temp distribution :-



Heat Conduction Through Variable Area :-

Assumptions :-

1) Steady state
2) One Dimensional Heat transfer
3) No Heat Generation
4) constant thermal conductivity
5) Homogeneous Material



→ The perfect insulated variable surface implies that the situation corresponds to one Dimensional conduction in the x-direction only for which

$$Q = -K A_x \cdot \frac{dT}{dx}$$

where A_x = cross-sectional area at any axial position 'x' from the thinner End of the rod.

∴ Radius at x :-

$$R_x = R_2 - \frac{R_2 - R_1}{L} x.$$

$$= R_2 - cx$$ where $c = \frac{R_2 - R_1}{L}$

$$\therefore A_x = \pi R_x^2$$

$$= \pi (R_2 - Cx)^2$$

$$\therefore Q = -K\pi (R_2 - Cx)^2 \frac{dT}{dx}.$$

seperating the variable and upon integration, we get

$$\int_0^L \frac{Q \cdot dx}{(R_2 - Cx)^2} = \int_{T_1}^{T_2} -K\pi \cdot dT$$

$$Q \int_0^L (R_2 - Cx)^{-2} \cdot dx = -K\pi \int_{T_1}^{T_2} dT$$

$$Q \left[\frac{(R_2 - Cx)^{-1}}{(-1) \times (-C)} \right]_0^L = -K\pi \left[T \right]_{T_1}^{T_2}$$

$$\frac{Q}{C} \left[\frac{1}{R_2 - CL} - \frac{1}{R_2} \right] = -K\pi (T_2 - T_1)$$

$$\frac{Q}{C} \left[\frac{1}{R_2 - CL} - \frac{1}{R_2} \right] = K\pi (T_1 - T_2)$$

substitute the value $C = \frac{R_2 - R_1}{L}$

$$Q \times \frac{L}{R_2 - R_1} \left[\frac{1}{R_2 - \left(\frac{R_2 - R_1}{\cancel{L}} \right) \cancel{L}} - \frac{1}{R_2} \right] = K\pi (T_1 - T_2)$$

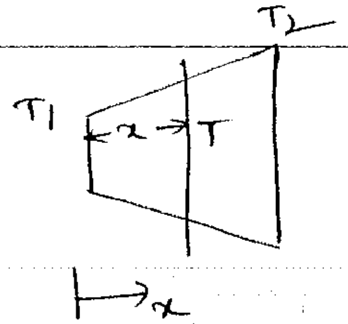
$$\frac{QL}{R_2 - R_1} \left[\frac{1}{R_1} - \frac{1}{R_2} \right] = K\pi (T_1 - T_2)$$

$$\frac{QL}{R_2-R_1}\left[\frac{R_2-R_1}{R_1R_2}\right] = K\pi(T_1-T_2)$$

$$\frac{QL}{R_1R_2} = K\pi(T_1-T_2)$$

$$\boxed{\therefore\ Q = \frac{K\pi R_1R_2(T_1-T_2)}{L}}$$

Temparature at any section 'x' from the inlet End:-



At
$x=0$
$T=T_1$

At
$x=x$
$T=T$

$$\therefore\ Q = -KA\cdot\frac{dT}{dx}$$

$$\int_0^x Q\cdot\frac{dx}{A} = \int_{T_1}^{T_2} -K\cdot dT$$

Conventional questions:-

1) A conical cylinder of length L and Radius R_1 & R_2 $(R_1<R_2)$ is fully insulated on outer surfaces and two ends are maintained at T_1 & T_2 $(T_1>T_2)$ considering 1-D steady heat flow. Derive the expression for Heat transfer, if $R_1 = 1.25cm$, $R_2 = 2.5cm$, $L = 20cm$, $T_1 = 227°C$, $T_2 = 27°C$, $K = 40W/mK$ And

1) steady state HT
2) Temp at mid plane

Data

$R_1 < R_2$ & $T_1 > T_2$

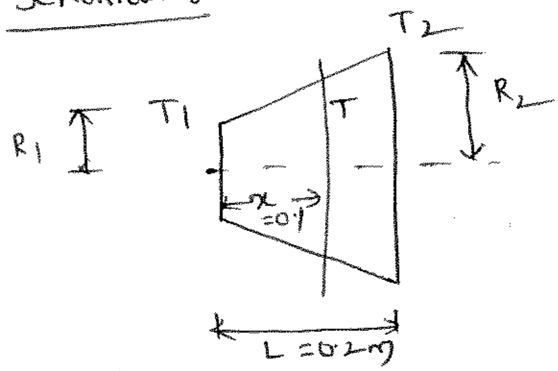
$R_1 = 1.25cm$, $R_2 = 2.5cm$, $L = 20cm$
$= 1.25 \times 10^{-2} m$, $= 2.5 \times 10^{-2} m$, $= 20 \times 10^{-2} m$

$T_1 = 227°C$, $T_2 = 27°C$, $k = 40 w/mk$

Find:-

1) Derive expression
2) $Q = ?$
3) $T_{mid} = ?$

schematic:-



Assumptions:-

1) steady state
2) 1-D
3) No q_g
4) $K = const$
5) Homogenious material

Sol

1) As previous derivation

2) $$Q = \frac{K \pi R_1 R_2 (T_1 - T_2)}{L} \Rightarrow Q = \frac{40 \times \pi \times 1.25 \times 2.5 (227 - 27)}{0.2}$$

$$Q = 39.26 W.$$

2) T_{mid} =?

$$Q = -kA \cdot \frac{dT}{dx}$$

$$\int_0^{0.1} Q \cdot \frac{dx}{A} = \int_{227}^{T} -k \cdot dT$$

$$A = \pi (R_1 + Cx)^2$$

$$C = \frac{R_2 - R_1}{L} \Rightarrow \frac{2.5 - 1.25}{20}$$

$$C = 0.0625$$

$$Q \int_0^{0.1} \frac{dx}{\pi (R_1 + Cx)^2} = -k(T - 227)$$

$$\frac{39.26}{\pi C}\left[\frac{-1}{R_1 + Cx}\right]_0^{0.1} = 40(227 - T)$$

$$\frac{-39.26}{\pi \times 0.0625}\left[\frac{1}{R_1 + C_1(x)} - \frac{1}{R_1}\right] = 40(227 - T)$$

$$\frac{-39.26}{\pi \times 0.0625}\left[\frac{1}{\quad}\right.$$

$$\therefore T = 93.6^{\circ}C$$

2) A structural support has a shape as shown in fig. It's Length is 0.2m & it's area varies with x as $A = \frac{\pi}{4} x^3$ it's circumference is perfectly insulated and thermal conductivity varies with temp as $k = 14.695(1 + 0.00102\,08T)$

where 'T' is °C and k is in w/m°c. what is the steady state Heat transfer. If two ends are Maintained at 400°C & 150°C and also find the temp at Mid plane.

Data :-

$L = 0.2m$, $A = \frac{\pi}{4} x^2$

$k = 14.695 (1 + 0.00102087)$

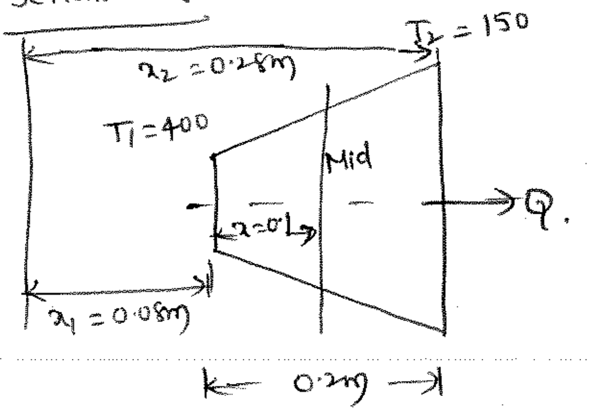
$T_1 = 400°C$, $T_2 = 150°C$.

Find :-

1) $Q = ?$

2) $T_{mid} = ?$

schematic :-



Assumptions :-

1) steady state 2) 1-D 3) No q_g 4) K = Const 5) Homogeneous

Sol

1) $Q = -KA \frac{dT}{dx}$

$$\int_{x_1}^{x_2} Q \cdot \frac{dx}{A} = \int_{T_1}^{T_2} -K \cdot dT$$

$$Q\int_{x_1}^{x_2} \frac{dx}{\pi x^2}\times 4 = \int_{T_1}^{T_2} 14.695(1+0.0010208T)\cdot dT$$

$$\frac{4Q}{\pi}\left[\frac{-1}{2x^2}\right]_{0.08}^{0.28} = -14.695\left[\frac{T+0.0010208T^2}{2}\right]_{400}^{150}$$

$$\frac{4Q}{\pi}\left[\frac{-1}{2(0.28)^2}-\frac{1}{2(0.08)^2}\right] = -14.695\left[\frac{150+0.0010208(150)^2}{2} - \frac{400+0.0010208(400)^2}{2}\right]$$

$$\therefore\ Q = 51.5W$$

2) T_{mid}

$$\int_{x_1}^{x_{mid}} \frac{dx\cdot 4}{\pi x^2} = -\int_{T_1}^{T_{mid}} 14.695(1+0.0010208T)\,dT$$

$$\frac{4\times 51.5}{\pi}\int_{0.08}^{0.18}\frac{1}{x^2}\cdot dx = -14.695\int_{400}^{T}(1+0.0010208T)\,dT$$

$$\therefore\ T = 184.5^\circ C$$

Note :- temperature profile is not linear.

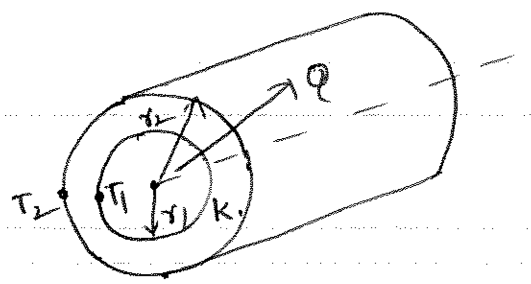
2) Long Hollow cylinder :-

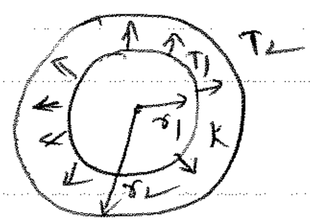
→ The heat ~~transfer~~ through ~~a~~ the pipe is in the normal direction to the pipe surface and no significant heat transfer takes place in the pipe in other directions.

→ ~~If the~~ The temp of the pipe in ~~this~~ this case depends on one direction only (the radial r-direction) and can be expressed as $T = T(r)$.

→ Consider a long hollow cylinder, the inner surface $r = r_1$ is kept at temperature T_1 & outer surface at $r = r_2$ is kept at temperature T_2. & There is no heat Generation and the thermal conductivity ~~is~~ of the solid is kept constant.

Steady state one Dimensional Heat Conduction in a hollow cylinder without Heat generation :-





Assumptions :-

1) steady state
2) One Dimensional Heat Conduction
3) No Heat Generation
4) Constant Thermal conductivity
5) Homogenious Material

∴ The fundamental Equation for cylindrical coordinates in 3-D is.

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial T}{\partial r} + \underbrace{\cancel{\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}}}_{\text{1-D}} + \underbrace{\cancel{\frac{\partial^2 T}{\partial z^2}}}_{\text{1-D}} + \underbrace{\cancel{\frac{q_g}{k}}}_{\text{No } q_g} = \underbrace{\cancel{\frac{1}{\alpha}\frac{\partial T}{\partial t}}}_{\text{steady state}}$$

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial T}{\partial r} = 0.$$

(or)

$$\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\cdot\frac{\partial T}{\partial r}\right) = 0.$$

(or)

$$\frac{\partial}{\partial r}\left(r\cdot\frac{\partial T}{\partial r}\right) = 0$$

Also we can write as following General Equation as ~~discussed~~ early

$$\frac{d}{dx}\left[x^n \cdot \frac{dT(x)}{dx}\right] = 0$$

$x = r$ & $n = 1$ → for cylindrical coordinates

$$\therefore \frac{d}{dr}\left[r\cdot\frac{dT(r)}{dr}\right] = 0.$$

where $T = T(r)$ is the function of r-direction

Intigrating

$$r\cdot\frac{dT}{dr} = C_1$$

$$\frac{dT}{dr} = \frac{C_1}{r_1}.$$

Again integrating

~~T(r) = c_1~~

$$\boxed{T = c_1 \ln r + c_2} \quad — ①$$

Now using ~~Inn~~ Inner (first) Boundary conditions

At, $r = r_1$, $T = T_1$ → Inner ~~side~~ surface

$$T_1 = c_1 \ln r_1 + c_2 \quad — ②$$

Now using secondary (outer) Boundary conditions

At, $r = r_2$, $T = T_2$ → outer surface

$$T_2 = c_1 \ln r_2 + c_2 \quad — ③$$

~~From~~ solving equations ② & ③

$$T_1 = c_1 \ln r_1 + \cancel{c_2}$$

$$\underline{-T_2 = -c_1 \ln r_2 - \cancel{c_2}}$$

$$T_1 - T_2 = c_1 \ln\left[\frac{r_1}{r_2}\right]$$

$$\boxed{c_1 = \frac{T_1 - T_2}{\ln\left[\frac{r_1}{r_2}\right]}}$$

$$\therefore T_1 = c_1 \ln r_1 + c_2$$

$$T_1 = \frac{T_1 - T_2}{\ln\left[\frac{r_1}{r_2}\right]} . \ln r_1 + c_2$$

$$\boxed{c_2 = T_1 - \frac{(T_1 - T_2)}{\ln\left(\frac{r_1}{r_2}\right)} \ln r_1}$$

$$\therefore \quad T = C_1 \ln r + C_2$$

$$T = \frac{(T_1 - T_2)}{\ln\left[\frac{r_1}{r_2}\right]} \ln r + T_1 - \frac{(T_1 - T_2)}{\ln\left(\frac{r_1}{r_2}\right)} \ln r_1$$

$$\boxed{\frac{T - T_1}{T_1 - T_2} = \frac{\ln\left[\frac{r}{r_1}\right]}{\ln\left[\frac{r_1}{r_2}\right]}}$$

Heat transfer :-

$$Q = -KA \cdot \frac{dT}{dr}$$

where $A = 2\pi r L$ (surface area)

$$Q = -k \cdot 2\pi r L \cdot \frac{dT}{dr}$$

$$Q = -k \cdot 2\pi \cancel{r} L \frac{C_1}{\cancel{r}} \qquad \Big| \because \frac{dT}{dr} = \frac{C_1}{r}$$

$$Q = -K \cdot 2\pi L \, C_1$$

$$Q = -K \cdot 2\pi L \left[\frac{T_1 - T_2}{\ln\left(\frac{r_1}{r_2}\right)}\right]$$

$$\therefore \boxed{Q = \frac{2\pi K (T_1 - T_2)}{\ln\left[\frac{r_2}{r_1}\right]}}$$

Electrical Analogy (concept of Thermal resistance) :-

$$\Delta T = Q \cdot R$$

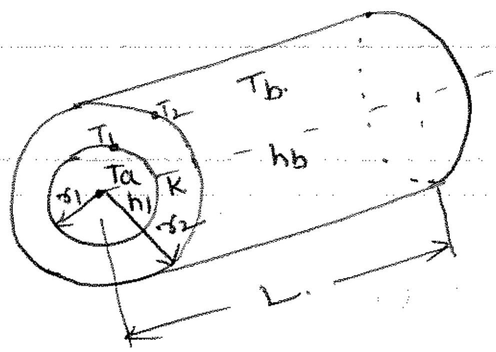
$$\because \quad Q = \frac{2\pi K L \overbrace{(T_1 - T_2)}^{\Delta T}}{\ln\left(\frac{r_2}{r_1}\right)}$$

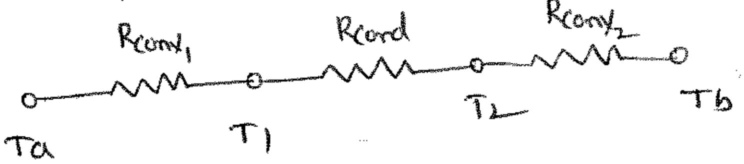
$$\Delta T = Q\left[\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi K L}\right]$$

$$\Delta T = Q\,R_{th}$$

$$\therefore \boxed{R_{th} = \frac{\ln\left[\frac{r_2}{r_1}\right]}{2\pi K L}} \rightarrow$$ Thermal Resistance for conduction in cylinder

Hollow cylinder subjected to convection heat transfer at inner and outer surfaces :-





$$\Delta T = Q R$$

$$Q = \frac{\Delta T}{R} \Rightarrow \boxed{Q = \frac{T_a - T_b}{\Sigma R_{th}}}$$

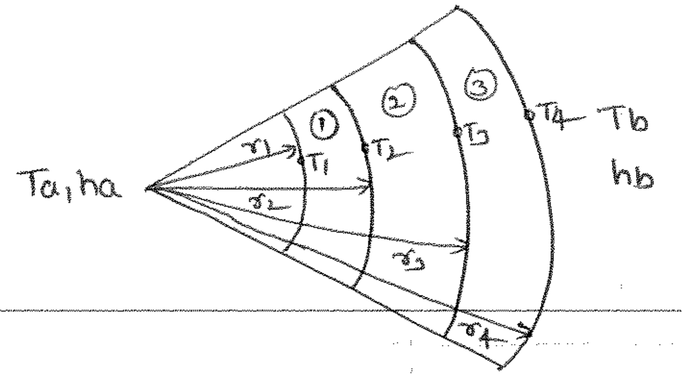
$$\Sigma R_{th} = R_{conv_1} + R_{cond} + R_{conv_2}$$

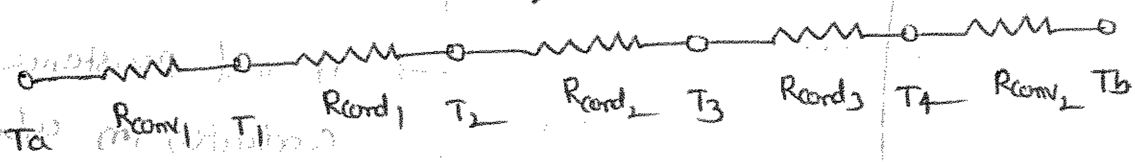
$$= \frac{1}{h_a(2\pi r_1 L)} + \frac{\ln(r_2/r_2)}{2\pi K L} + \frac{1}{h_b(2\pi r_2 L)}$$

$R_{conv} = \frac{1}{hA}$

$A = 2\pi r L$

Composit cylinder :-





$$\Delta T = Q R$$

$$Q = \frac{\Delta T}{R} \Rightarrow \boxed{Q = \frac{T_a - T_b}{\Sigma R_{th}}}$$

$$\Sigma R_{th} = R_{conv_1} + R_{cond_1} + R_{cond_2} + R_{cond_3} + R_{conv_2}$$

$$= \frac{1}{h_a(2\pi r_1 L)} + \frac{\ln(r_2/r_1)}{2\pi K_1 L} + \frac{\ln(r_3/r_2)}{2\pi K_2 L} + \frac{\ln(r_4/r_3)}{2\pi K_3 L} + \frac{1}{h_b(2\pi r_4 L)}$$

overal Heat transfer co-efficient (U) :-

$(\cancel{\Delta T} = QR \Rightarrow \cancel{Q = \Delta T}$

$$(\Delta T)_{overal} = Q\,\Sigma R_{th} \Rightarrow Q = \frac{\Delta T}{\Sigma R_{th}}$$

$Q = U_1 A_1 (\Delta T)_{overal}$ — | Like w.r.s $Q = \cancel{hA\Delta T}\ hA\,\Delta T$

$$\therefore \frac{(\Delta T)_{overall}}{\Sigma R_{th}} = U_1 A_1 (\Delta T)_{overall}$$

$$U_1 = \frac{1}{A_1 \Sigma R_{th}} \qquad \therefore A_1 = 2\pi r_1 L.$$

$$U_1 = \frac{1}{2\pi r_1 L [\Sigma R_{th}]}$$

$$U_1 = \frac{1}{2\pi r_1 L\left[\frac{1}{h_a(2\pi r_1 L)} + \frac{\ln(r_2/r_1)}{2\pi k_1 L} + \frac{\ln(r_3/r_2)}{2\pi k_2 L} + \frac{\ln(r_4/r_3)}{2\pi k_3 L} + \frac{1}{h_b(2\pi r_4 L)}\right]}$$

$$\boxed{U_1 = \frac{1}{\frac{1}{h_a} + \frac{r_1 \ln(r_2/r_1)}{k_1} + \frac{r_1 \ln(r_3/r_2)}{k_2} + \frac{r_1 \ln(r_4/r_3)}{k_3} + \frac{r_1}{r_4 h_b}}}$$

Like wise

$$U_1 A_1 = U_2 A_2 = U_3 A_3 = U_4 A_4.$$

$$\therefore \boxed{U_i A_i = U_o A_o}$$

$$U_i \cancel{2\pi} r_i \cancel{L} = U_o \cancel{2\pi} r_o \cancel{L}$$

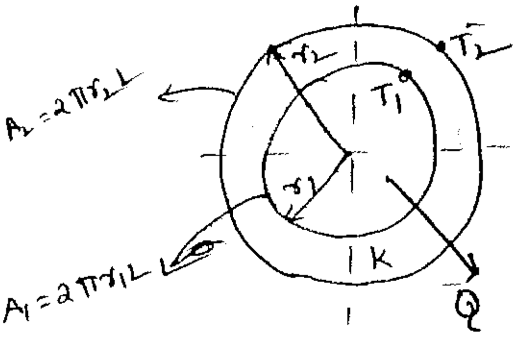
$$\boxed{U_i r_i = U_o r_o} \rightarrow \text{valid for cylinder only}$$

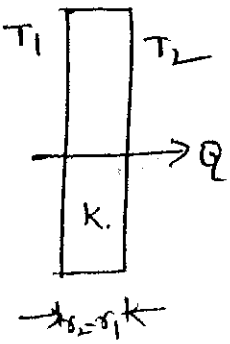
o - outer
i - inner

Logarithmic Mean Area :-

→ This Approach can be used to transform a cylinder into a equivalent slab of thickness $(r_2 - r_1)$ as shown in fig

→ This area of the ~~slab~~ plane slab which gives the same heat transfer for the same thermal conductivity and for same thickness of cylinder





cylinder ↓ Equivalent slab ↓

$$Q = \frac{2\pi K L (T_1 - T_2)}{\ln (r_2/r_1)} \quad \text{--- (1)}$$

$$\Delta T = Q R$$

$$T_1 - T_2 = Q \frac{r_2 - r_1}{K A_m} \qquad \left| \; L = \frac{L}{KA} \right.$$

$$\boxed{Q = \frac{(T_1 - T_2) K A_m}{r_2 - r_1}} \quad \text{--- (2)}$$

↓ Heat flow for Hollow cylinder can be written above

Equating (1) & (2)

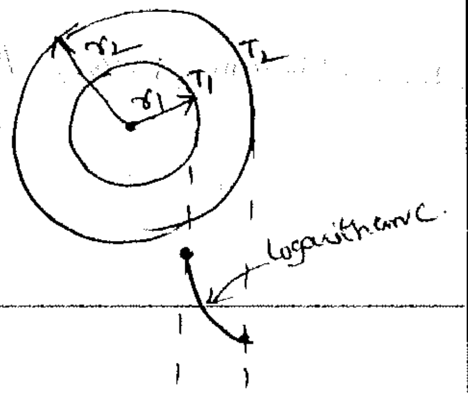
$$\frac{2\pi \cancel{K} L \cancel{(T_1 - T_2)}}{\ln (r_2/r_1)} = \frac{\cancel{(T_1 - T_2)} \cancel{K} A_m}{(r_2 - r_1)}$$

$$\boxed{A_m = \frac{2\pi L (r_2 - r_1)}{\ln (r_2/r_1)}}$$

$$A_m = \frac{2\pi r_2 L - 2\pi r_1 L}{\ln \left[\frac{2\pi r_2 L}{2\pi r_1 L} \right]}$$

$$\boxed{A_m = \frac{A_2 - A_1}{\ln \left(\frac{A_2}{A_1} \right)}}$$

Temperature profile :-



$\therefore \; Q =$

Conventional Questions :-

1) A cylindrical Insulation for a steam pipe has Inside radius of 6cm and outside radious of 8cm, Thermal conductivity is equal to 0.85 w/mK. The Inside surface of the insulation is at temp of 430°C and outside surface temp is of 30°C. Determine

1) Heat loss per meter length of this Insulation
2) Temparature at the Mid thickness of insulation
3) Draw temparature profile.

Data

$r_1 = 6m \quad , \quad r_2 = 8\,cm$

$K_{ins} = 0.85\, w/mK$

$T_i = T_1 = 430°C \quad , T_0 = T_2 = 30°C$

Find

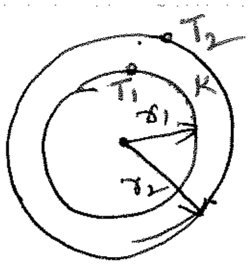
1) Heat loss/m length $(Q = ?)$
2) $T_{mid} = ?$
3) Temp profile

Sol

1)

$\Delta T = Q R_{th}$

$R_{th} = \dfrac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi K L}$



$\therefore \; 430 - 30 = Q\left[\dfrac{\ln(8/6)}{2 \times \pi \times 0.85 \times 1}\right]$

$Q = 4368\; w/m$

2)
Temparature at any position in cylinder is

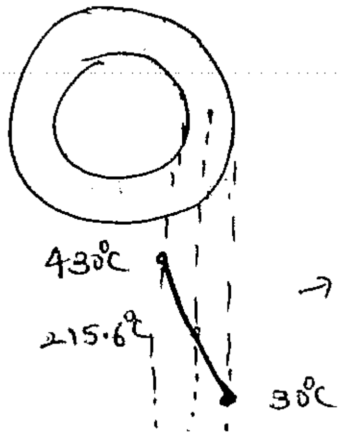
$$\frac{T - T_1}{T_1 - T_2} = \frac{\ln(r/r_1)}{\ln(r_1/r_2)}$$

At middle
$r = 6 + \frac{2}{1}$
$= 7mm$

$$\frac{T - 430}{430 - 30} = \frac{\ln(7/6)}{\ln(6/8)}$$

$$\therefore T_{mid} = 215.6^\circ C$$

3) Temp profile



→ This is not a straight line. It is a curve

2) A 10cm outside dia pipe carrying saturated steam at a temp of 195°C is Lagged to 20cm dia with Magnesia and further Lagged with asbestos to 25cm dia. If the temp at outer surface of asbestos is 20°C. Find the mass of steam condensed in 8 hours on a 100m length pipe. Take $K_{magnesia} = 0.07 W/mK$, $K_{asb} = 0.082 W/mK$, neglect the thermal Resistance of pipe material and enthalpy of vaporisation is 1951 KJ/kg.

~~r_1~~ $d_1 = 10cm$ $\rightarrow T_1 = 195^\circ C$

$r_1 = 5cm$

$d_2 = 20cm$

$r_2 = 10cm$

$d_3 = 25cm$ $\rightarrow T_3 = 20^\circ C$

$r_3 = 12.5cm$

time $(t) = 8$ hours

$= 8 \times 60 = 480$ min

$= 480 \times 60 = 28{,}800$ sec

Length of pipe $(L) = 100m$

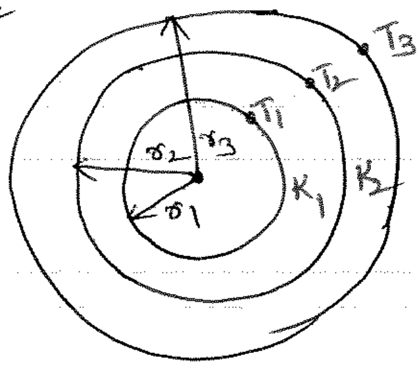
$K_{mag} = 0.07\ W/mK$, $K_{asb} = 0.082\ W/mK$

$h_{fg} = 1951\ KJ/kg$

Find

1) Mass of steam condensed $(m = ?)$

Sol



~~Heat transfer~~

$Q = \dot{m}\, h_{fg}$

$\dot{m} = \dfrac{Q}{h_{fg}}$

$L_1 = L_2$

from $\Delta T = QR$

$$T_1 - T_3 = Q\left[\frac{\ln(r_2/r_1)}{2\pi K_1 L_1} + \frac{\ln(r_3/r_2)}{2\pi K_2 L_2}\right]$$

$$195 - 20 = Q\left[\frac{\ln(10/5)}{2\times\pi\times0.07\times100} + \frac{\ln(12.5/10)}{2\times\pi\times0.082\times100}\right]$$

$$\therefore Q = 8710.5\ W$$

$Q = 8710.5\ J/s$

for '1' sec heat loss $= 8710.5\ J$

for 8 hours heat loss $= 8710.5 \times 28{,}800$

$= 250.86 \times 10^6\ J$

$= 250.86 \times 10^3\ kJ$

$\therefore$ mass of steam condensed $(\dot{m}) = \dfrac{Q}{h_{fg}}$

$= \dfrac{250.86 \times 10^3}{1951}$

$\dot{m} = 128.5\ kg$

3) A Metal ($K = 45\ W/mK$) steam pipe of 5cm inner dia and 6.5cm outer dia is Lagged with 2.7cm Radial thickness of insulation having thermal conductivity of 1.1 W/mK. convective heat transfer co-efficient on the inside and outside surfaces are $h_i = 4650\ W/m^2K$ and $h_o = 11.5\ W/m^2K$ respectively, if the steam temp is 200°C and the ambient temp is 25°C. calculate

1) Heat loss per meter Length
2) temp at interfaces
3) over heat transfer co-effients based on inside and outside surfaces.

Data

$K_{pipe} = 45\ W/mK$, $d_1 = 5cm$ $\quad d_2 = 6.5cm$

$r_1 = 2.5cm$ $\quad r_2 = 3.25cm$

$r_3 = r_2 + t_1$, $K_{ins} = 1.1\ W/mK$

$= 3.25 + 2.75$,

$= 6\ cm$

$h_i = 4650\ W/m^2K$, $h_o = 11.5\ W/m^2K$

$T_{steam} = T_i = \cancel{T_{a}} = 200^\circ C$

$T_\infty = 25^\circ C. = T_0$

Find

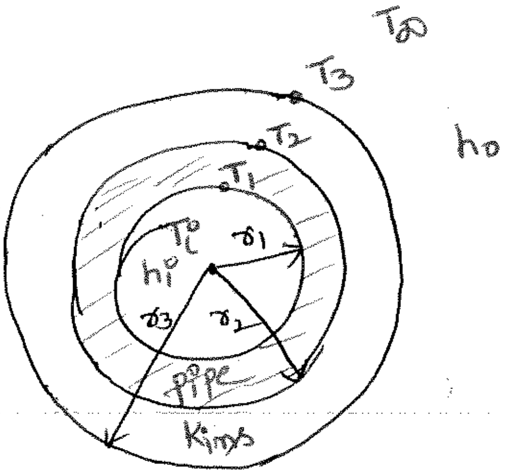
1) Heat loss per meter length $(Q = ?)$

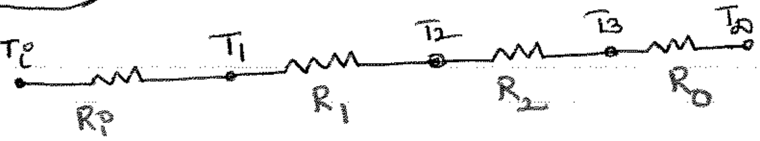
2) Temperatures at Interfaces ~~$(T_1 \& T_2, T_3 = ?)$~~

$(T_1, T_2 \& T_3 = ?)$

3) $U_i = ?$

Sol





1) $\Delta T = Q R_{effect}$

$R_{effect} = R_i + R_1 + R_2 + R_0$

$$R_i = \frac{1}{h_i A_i} = \frac{1}{4650 \times 2\pi \times (2.5 \times 10^{-2}) \times 1} = 1.369 \times 10^{-3}\ K/W$$

$$R_1 = \frac{\ln(r_2/r_1)}{2\pi K_p L_2} = \frac{\ln(3.25/2.5)}{2\times\pi\times 45\times 1} = 9.279\times10^{-4}\ K/W$$

$$R_2 = \frac{\ln(r_3/r_2)}{2\pi K_{ins} L_3} = \frac{\ln(6/3.25)}{2\times\pi\times 1.1\times 1} = 0.0887\ K/W.$$

$$R_0 = \frac{1}{h_0 A_0} = \frac{1}{11.5\times 2\times\pi\times(6\times10^{-2})\times 1} = 0.2307\ K/W.$$

$\therefore$ ~~Reff~~

$$\Delta T = Q R_{eff}$$

$$200 - 25 = Q\left[1.369\times10^{-3} + 9.279\times10^{-4} + 0.0887 + 0.2307\right]$$

$$\therefore Q = 545.17\ W/m$$

2)

$$Q = hA\Delta T$$

$$\Delta T = Q\left[\frac{1}{hA}\right] \Rightarrow [\Delta T = QR]$$

$$T_i - T_1 = 545.17\left[\frac{1}{4650\times 2\times\pi\times(2.5\times10^{-2})\times 1}\right]$$

$$T_1 = 199.25^\circ C$$

$$\Delta T = QR$$

$$T_1 - T_2 = Q\left[\frac{\ln(r_2/r_1)}{2\pi K_p L}\right]$$

$$199.25 - T_2 = 545.17\left[\frac{\ln(3.25/2.5)}{2\times\pi\times 45\times 1}\right]$$

$$T_2 = 198.74^\circ C$$

$$\Delta T = QR$$

$$T_2 - T_3 = Q\left[\frac{\ln(r_3/r_2)}{2\pi K_{ins} L}\right]$$

$$198.74 - T_3 = 545.1\left[\frac{\ln(6/3.25)}{2\times\pi\times 1.1\times 1}\right]$$

$$T_3 = 150.76^\circ C.$$

3)

$$Q = UA\Delta T = U_i A_i \Delta T = U_0 A_0 \Delta T$$

~~545.1 = U_i × 2πx~~

$$\Delta T = (T_i - T_0)$$

$$\therefore Q = U_i A_i \Delta T$$

$$545.1 = U_i \times 2\times\pi(2.5\times10^{-2})\times 1\ (200-25)$$

$$U_i = 19.83\ W/m^2K$$

4) A 160mm dia pipe carrying saturated steam is covered by A layer of lagging of thickness 40mm (K = 0.8 W/mK) Latter on an extra layer of lagging of 10mm thickness (K = 0.12 W/mK) is added, if the surrounding temp remains constant and heat transfer co-efficient for both lagging material 10 W/m²K. Determine

1) The % change in heat transfer due to extra lagging.

$d_1 = 160mm$ $\quad r_2 = r_1 + t_1$

$r_1 = 80mm$ $\quad = 80 + 40$ $\rightarrow K_1 = 0.8\, W/mK.$

$= 120mm$

$r_3 = r_2 + t_2$

$= 120 + 10$ $\rightarrow K_2 = 0.12 W/mK$

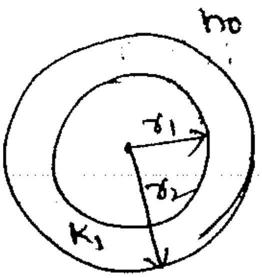
$= 130mm$

$h_0 = 10 W/m^2K$

find

% change in heat transfer due to extra lagging

case 1 :-

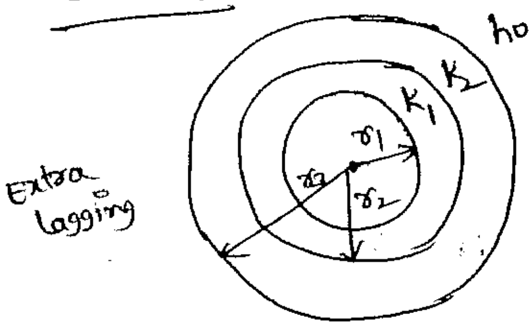


$\Delta T = QR$

$$\Delta T = Q_1\left[\frac{\ln\left(\frac{120}{80}\right)}{2\times\pi\times 0.8\times 1} + \frac{1}{10\times 2\pi\times 0.12\times 1}\right]$$

$\Delta T = 0.2136\, Q_1$ — (1)

case 2 :-



$\Delta T = QR$

$$\Delta T = Q_2\left[\frac{\ln(120/80)}{2\times\pi\times 0.8\times 1} + \frac{\ln(130/120)}{2\times\pi\times 0.12\times 1} + \frac{1}{10\times 2\pi\times 0.13\times 1}\right]$$

$\Delta T = 0.309\, Q_2$ — (2)

The temperature difference is same [remains const]

$\therefore \Delta T_{case 1} = \Delta T_{case 2}$

$0.2136\, Q_1 = 0.309\, Q_2$

$Q_2 = 0.6897\, Q_1$

$\therefore$ % change in HT $= \frac{Q_2 - Q_1}{Q_1}\times 100 = \frac{0.6897 Q_1 - Q_1}{Q_1} \times 100$

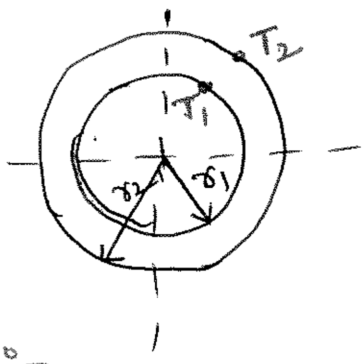
$= -0.3106 = -31.06\%$

3) ~~Hollow~~ Hollow sphere :-

→ consider a hollow sphere of inner radius of r_1 and outer radius of r_2 as shown in fig below.

→ The inner and outer ~~temperatures~~ surfaces are maintained uniform temperatures T_1 and T_2 respectively

Steady state one Dimensional Heat conduction in a hollow sphere with out Heat generation :-



Assumptions :-

1) steady state
2) one Dimensional
3) No Heat generation
4) constant thermal conductivity
5) Homogenious Material

∴ The fundamental Equation for spherical coordinates in 3-D is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \cancel{\frac{1}{r^2}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)}^{\text{1-D}} + \cancel{\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}}^{\text{1-D}} + \cancel{\frac{q_g}{k}}^{\text{No } q_g} = \frac{1}{\alpha}\cancel{\frac{\partial T}{\partial t}}^{\text{steady state}}$$

~~Step~~

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\cdot\frac{\partial T}{\partial r}\right)=0$$

$$\frac{\partial}{\partial r}\left(r^2\cdot\frac{\partial T}{\partial r}\right)=0.$$

$$\frac{d}{dr}\left(r^2\cdot\frac{dT(r)}{dr}\right)=0$$

integrating the equation w.r.t 'r'

$$\frac{r^2\, dT(r)}{dr}=C_1$$

$$\frac{dT(r)}{dr}=\frac{C_1}{r^2}$$

Again integrating

$$\boxed{T(r)=\frac{-C_1}{r}+C_2} \quad —(1)$$

Now using Inner (first) boundary conditions

At, $r=r_1$, $T=T_1$ → Inner surface

$$T_1=\frac{-C_1}{r_1}+C_2 \quad —(2)$$

Now using ~~seco~~ outer (secondary) Boundary conditions

At $r=r_2$, $T=T_2$ → outer surface

$$T_2=\frac{-C_1}{r_2}+C_2 \quad —(3)$$

Solving equation (2) and (3)

$$\cancel{T_1=\frac{-C_1}{r}+C_2}$$

$$\cancel{T_2=\frac{-C_1}{r}}$$

$$T_1 = \frac{-C_1}{r_1} + C_2$$

$$T_2 = \frac{-C_1}{r_2} + C_2$$

$$- \quad + \quad -$$

$$T_1 - T_2 = C_1\left[\frac{1}{r_2} - \frac{1}{r_1}\right] \qquad T_1 - T_2 = C_1 \; r_2 - r$$

$$\boxed{C_1 = \frac{(T_1 - T_2)\, r_1 r_2}{r_2 - r_1}}$$

$$\because \quad T_1 = \frac{-C_1}{r_1} + C_2$$

$$T_1 = -\frac{(T_1 - T_2)\, r_1 r_2}{r_1 (r_2 - r_1)} + C_2$$

$$\boxed{C_2 = \frac{r_2 T_2 - r_1 T_1}{r_2 - r_1}}$$

Substituting C_1 and C_2 in equation (1)

$$\boxed{T(r) = \frac{r_1 r_2}{r}\left(\frac{T_1 - T_2}{r_2 - r_1}\right) + \frac{r_2 T_2 - r_1 T_1}{r_2 - r_1}} \rightarrow (4)$$

Di~~fferentiating the equation w.r.t r we get~~

~~$\frac{dT(r)}{}$~~

Heat transfer :-

$$Q = -KA \cdot \frac{dT}{dr}$$

where $A = 4\pi r^2$

$$Q = -K \cdot 4\pi r^2 \frac{dT}{dr}$$

Differentiating Equation (4) we get

$$\frac{dT(r)}{dr} = -\frac{1}{r^2} \frac{(r_1 r_2)(T_1 - T_2)}{r_2 - r_1}$$

$$\therefore Q = -K \cdot 4\pi r^2 \left[\frac{-1}{r^2} \frac{(r_1 r_1)(T_1 - T_2)}{r_2 - r_1} \right]$$

$$Q = \frac{K}{\cancel{r^2}} (r_1 r_1) \left(\frac{T_1 - T_2}{r_2 - r_1} \right) 4\pi \cancel{r^2}$$

$$\boxed{Q = \frac{4\pi K r_1 r_2 (T_1 - T_2)}{r_2 - r_1}}$$

Concept of Electrical Analogy (Thermal Resistance) :-

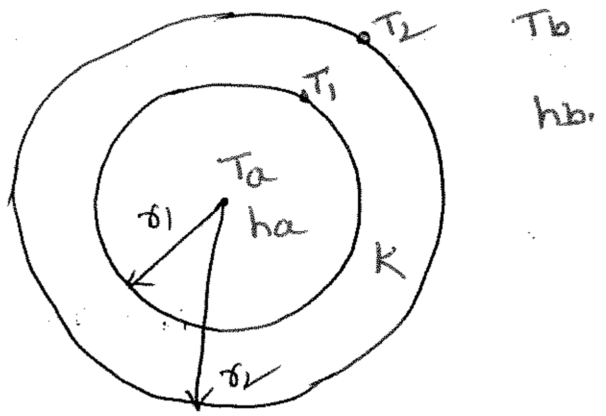
$$\Delta T = QR$$

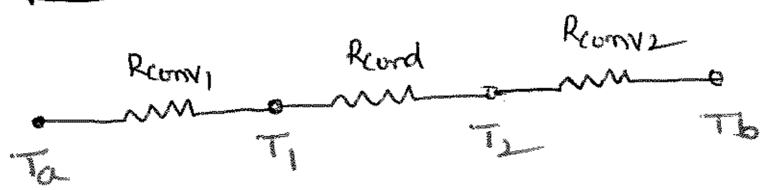
$$\because Q = \frac{4\pi K r_1 r_2 \overbrace{(T_1 - T_2)}^{\Delta T}}{r_2 - r_1}$$

$$\Delta T = Q \left[\frac{r_2 - r_1}{4\pi K r_1 r_2} \right]$$

$$\boxed{R_{th} = \frac{r_2 - r_1}{4\pi k r_1 r_2}} \rightarrow$$ Thermal Resistance for conduction in a sphere

Hollow sphere subjected to convection heat transfer at inner and outer surfaces :-





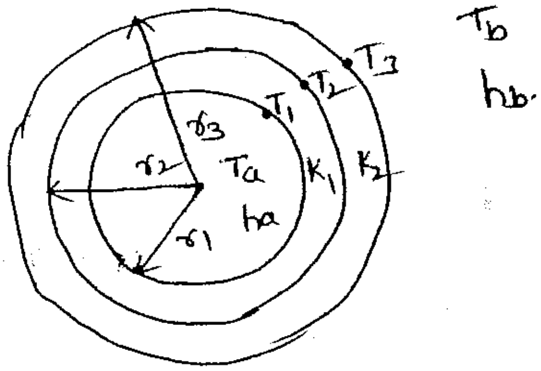
$$\Delta T = QR$$

$$Q = \frac{\Delta T}{R} \Rightarrow \boxed{Q = \frac{T_a - T_b}{\Sigma R_{th}}}$$

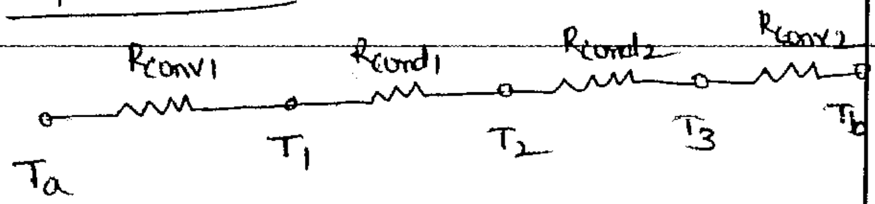
$$\Sigma R_{th} = R_{conv1} + R_{cond} + R_{conv2}$$

$$= \frac{1}{h_a \times 4\pi r_1^2} + \frac{r_2 - r_1}{4\pi k r_1 r_2} + \frac{1}{h_b \times 4\pi r_2^2}$$

Multiple layer Hollow sphere :-



Composite sphere



$$\Delta T = Q(R_{eff})$$

$$T_a - T_b = Q R_{eff}$$

$$R_{eff} = R_{conv1} + R_{cond1} + R_{cond2} + R_{conv2}$$

$$= \frac{1}{h_a 4\pi r_1^2} + \frac{r_2 - r_1}{4\pi r_1 r_2 K} + \frac{r_3 - r_2}{4\pi r_2 r_3 K} + \frac{1}{h_b 4\pi r_3^2}$$

overal Heat Transfer co-Efficient :-

$$Q = UA\Delta T$$

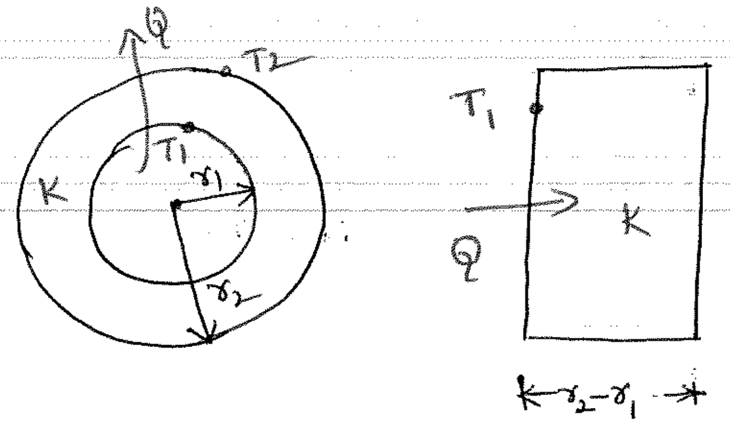
$$Q = U_i A_i \cancel{\Delta T} = U_0 A_0 \cancel{\Delta T}$$

$$U_i A_i = U_0 A_0$$

$$U_i 4\pi r_i^2 = U_0 4\pi r_0^2$$

$$\boxed{U_i r_i^2 = U_0 r_0^2}$$

Concept of Geometric Mean Area :-



$$\Delta T = QR$$

$$T_1 - T_2 = Q\left[\frac{r_2 - r_1}{KA_m}\right]$$

$$Q = \frac{KA_m(T_1 - T_2)}{r_2 - r_1}$$

Conduction in sphere = conduction in Equavilant slab

$$\frac{4\pi \cancel{K} r_1 r_2 \cancel{(T_1 - T_2)}}{\cancel{r_2 - r_1}} = \frac{\cancel{K}A_m \cancel{(T_1 - T_2)}}{\cancel{r_2 - r_2}}$$

$$4\pi r_1 r_2 = A_m$$

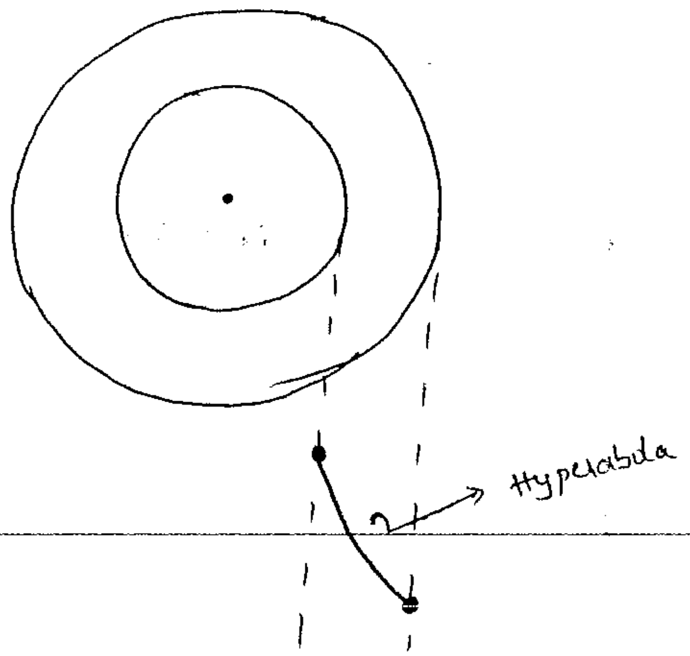
$$A_m = \sqrt{4\pi r_1^2 \cdot 4\pi r_2^2}$$

$$\boxed{A_m = \sqrt{A_1 \cdot A_2}}$$

" $\sqrt{\ }$ " is geometric mean

It is the Area of the plane slab which gives the same heat transfer for the same thermal conductivity and for same thickness of cylinder.

Temparature profile :-

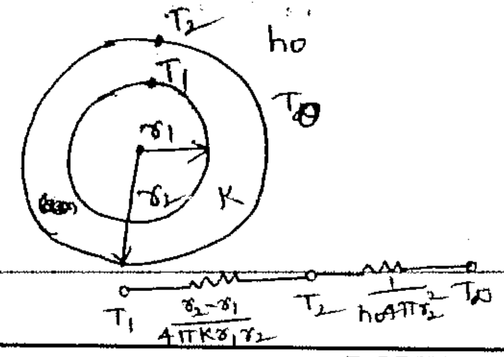


Conventional Questions :-

1) Consider an Al hollow sphere an inside radious $r_1 = 2cm$ and outside radious of $r_2 = 6cm$ and $K = 200 W/mK$. The Inside surface is kept at a temp of $100°C$ and outside surface loses heat by convection with $h = 80 W/m^2K$. into atmospheric air at a temp of $20°C$.

Determine
1) outside surface temp of sphere under steady state
2) Rate of heat transfer and sketch the temp distribution

Data



$r_1 = 2cm$

$r_2 = 6cm$

$K = 200 W/mK$ $\quad T_\infty = T_0 = 20°C$

$T_1 = 100°C$

$h = 80 W/m^2K$

$$\Delta T = QR$$

$$\underset{\cancel{100-20}}{T_1 - T_\infty} = Q\left[\frac{r_2 - r_1}{4\pi k r_1 r_2} + \frac{1}{h_o 4\pi r_2^2}\right]$$

$$100 - 20 = Q\left[\frac{6-2}{4\times\pi\times 200\times 2\times 6} + \frac{1}{80\times 4\pi\times (6\times 10^{-2})^2}\right]$$

$$Q = 276.7\,W$$

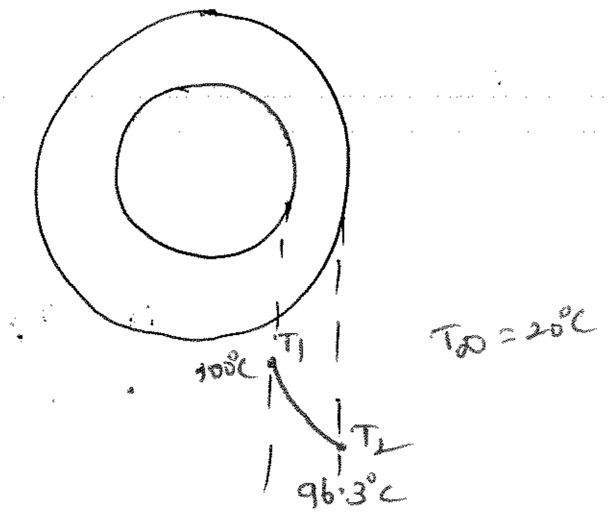
$$\therefore \Delta T = QR$$

$$T_1 - T_2 = Q\left[\frac{r_2 - r_1}{4\pi k r_1 r_2}\right]$$

$$100 - T_2 = 276.7\left[\frac{6-2}{4\times\pi\times 20\times 2\times 6}\right]$$

$$\underline{\underline{T_2 = 96.3^\circ C}}$$

<u>Temp profile</u>



2) A hollow spherical ~~form~~ is used to determine thermal conductivity of an insulating material. The Inner diameter is 50mm and outer diameter is 100mm. A 40W heater is placed inside and under steady conditions, the temp at 32 and 40mm ~~radii~~ radii were found to be 100°C and 70°C, respectively. Determine the thermal conductivity of the material. Also calculate the outside temp of sphere. If surrounding air is at 30°C, calculate convection heat transfer co-efficient over the surface.

Data

$d_1 = 50mm$ $\quad$ $d_2 = 100mm$

$r_1 = 25mm = 0.025m$ $\quad$ $r_2 = 50mm = 0.05m$

$Q = 40W$

$r_3 = 32mm = 0.032m$ $\quad$ $r_4 = 40mm = 0.04m$

$T_3 = 100°C.$ $\quad$ $T_4 = 70°C$

$T_\infty = 30°C.$

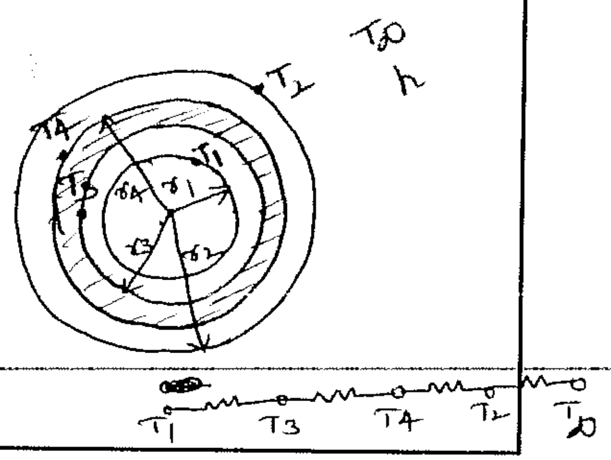
Find

1) Thermal conductivity of Insulating material $(K_{ins}) = ?$

2) outer temp of sphere $(T_2) = ?$

3) convection heat transfer coefficient $(h) = ?$

Sol



under steady condition rate of transfer rate.

1)

$$Q = \frac{4\pi k r_3 r_4 (T_3 - T_4)}{r_4 - r_3}$$

$$40 = \frac{4 \times \pi \times k \times 0.032 \times 0.04 (100-70)}{(\cancel{0.03}\ 0.04 - 0.032)}$$

$$\therefore K = \underline{0.663 W/mK}$$

2)

$$Q = \frac{4\pi k r_2 r_4 (T_4 - T_2)}{r_2 - r_4}$$

$$40 = \frac{4 \times \pi \times 0.663 \times \cancel{0.04}\ 0.05 \times 0.04 (70 - T_2)}{(0.05 - 0.04)}$$

$$T_2 = \underline{46^\circ C.}$$

3)

$$Q = hA\Delta T$$

$$= h 4\pi r_2^2 (T_2 - T_\infty)$$

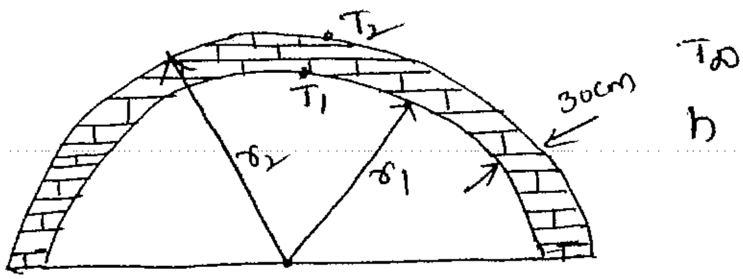
$$40 = h \times 4 \times \pi \times (0.05)^2 \times (46-30)$$

$$h = \underline{79.6 W/m^2K.}$$

3) A 7m diameter vertical Klin has a hemispherical dome at its top. The dome is made of 30cm thick layer of chrome brick ($K = 1.16\ W/mK$). During an operation, its inside surface temp is 900°C and outer surface ~~temp~~ is exposed to surrounding air at 30°C with heat transfer coefficient of $15\ W/m^2K$. calculate the outside surface temp of dome and the heat loss from the Klin.

Compare this heat loss with that would result from a flat dome made of same material and Klin operating under steady conditions.

Data



$d_1 = 7m$

$r_1 = 3.5m$

$r_2 = 3.5m + 30cm$
$= 3.8m$

$K = 1.16\ W/mK$

$T_1 = 900°C, \quad T_\infty = 30°C.$

$h = 15\ W/m^2K.$

Find

1) outside surface temp of dome ($T_2 = ?$)

2) Heat loss (Q) = ?

3) Heat loss from flat dome (Q_2) and % of heat ~~change~~ loss change

Sol under steady state condition

1) Heat conduction in dome = Heat convection from dome

$$\frac{1}{2}\left[\frac{4\pi K r_1 r_2 (T_1 - T_2)}{r_2 - r_1}\right] = \left[h\, 2\pi r_2^2 (T_2 - T_\infty)\right]$$

hemisphere Area

['1/2' is used ∵ it is hemisphere]

$$\frac{\cancel{4}\cancel{\pi} K r_1 r_2 (T_1 - T_2)}{r_2 - r_1} = h \cancel{\pi} r_2^2 (T_2 - T_\infty)$$

$$\frac{K r_1 (T_1 - T_2)}{r_2 - r_1} = h r_2 (T_2 - T_\infty)$$

$$\frac{1.16 \times 3.5 (900 - T_2)}{3.8 - 3.5} = 15 \times 3.8 (T_2 - 30)$$

$$T_2 = 196.92^\circ C$$

2) Heat loss from hemisphere dome

$$Q_1 = \frac{1}{2}\left[\frac{4\pi K r_1 r_2 (T_1 - T_2)}{r_2 - r_1}\right]$$

$$= \frac{1}{2}\left[\frac{4 \times \pi \times 1.16 \times 3.5 \times 3.8 (900 - 196.92)}{3.8 - 3.5}\right]$$

$$Q_1 = 227.18 \text{ KW}$$

3) If we use a flat top dome instead of hemisphere dome

$$\cancel{Q_2 = \frac{KA(T_1-T_2)}{L}}$$

Heat conduction = Heat convection

$$\frac{K\cancel{A}(T_1-T_2)}{L} = h\cancel{A}(T_2-T_\infty)$$

$$\frac{1.16\times(900-T_2)}{0.3} = 15(T_2-30)$$

$$T_2 = 208.3^\circ C$$

$$\therefore Q_2 = \frac{KA(T_1-T_2)}{L}$$

$$= \frac{K\cdot\pi r_1^2(T_1-T_2)}{L}$$

$$= \frac{1.16\times\pi\times3.5^2(900-208.3)}{0.3}$$

$$= 102.93\,KW$$

Mean Area

$A = \frac{\pi}{4}D^2$

$= \frac{\pi}{4}(2r)^2$

$= \frac{\pi}{\cancel{4}}\cancel{4}r^2$

$= \pi r^2$

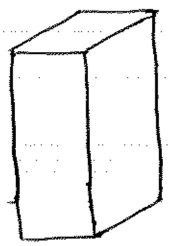
$\therefore$ % of Heat loss

$$\frac{227.18-102.93}{227.18} = 54.69\%$$

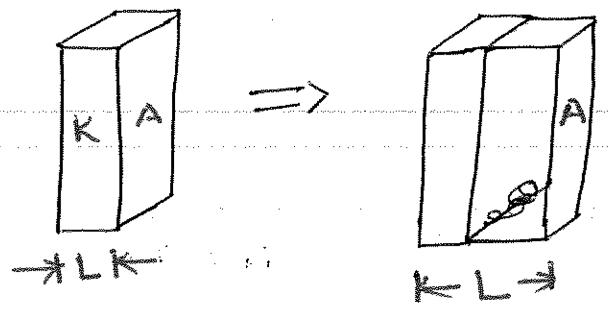
Critical Thickness of Insulation:-

→ In general perception (practice) that the addition of Insulation on a surface minimize the heat loss rate, a thickness of the Insulator lower the heat transfer rate.

1) plane slab:-



In case of plane slab there is no change in the surface area and therefore adding insulation would result in increase in conductive resistance $\left[R = \frac{L}{KA}\right]$, but the convective resistance remains same as the area is constant and hence with the addition of insultation the effective resistance increases and hence heat transfer decreases.



$\therefore \uparrow R_{cond} = \frac{L\uparrow}{KA}$

$R_{conv} = \frac{1}{hA} = \text{const}$

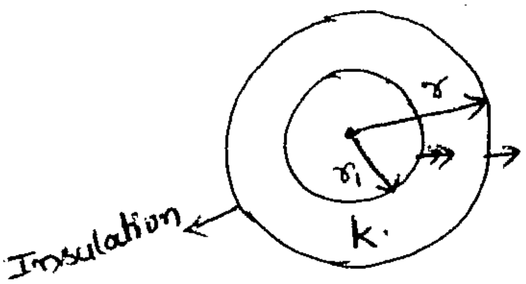
2) cylinder:-

→ For cylindrical pipes and sphere exposed to convection environment, the addition of Insulation, how ever is a different matter

→ The addition of insulation increase the conductive Resistance $\left[\frac{\ln(r_2/r_1)}{2\pi KL}\right]$ and decrease the convective Resistance $\left[R_{conv} = \frac{1}{h2\pi rL}\right]$ due to Increase the surface of exposure

→ The heat transfer rate may Increase (or) decrease depending on total effects ~~of~~ resistance

→ Consider a layer of insulation of thermal conductivity K, applied on a circular pipe of radious r_1. The Inner surface Temperature T_1 & outer surface ~~Temperature~~ is exposed to Environment T_∞ with heat transfer co-efficient h..



$$R_{cond} = \frac{\ln(r/r_1)}{2\pi KL}$$

$$R_{conv} = \frac{1}{h2\pi rL}$$

$$\Delta T = QR \Rightarrow QR = \text{const} \Rightarrow Q \propto \frac{1}{R}$$

$R_{total} = R_{cond} + R_{conv}$ [if Radiation HT neglects]

$$R_t = \frac{\ln\left(\frac{r}{r_1}\right)}{2\pi KL} + \frac{1}{h2\pi rL}$$

for Max (or) Min, differentiate above equation with "r" and equating to Zero

$$\frac{dR_t}{dr} = \frac{1}{2\pi kL}\left[\frac{r_1}{r_1}\cdot\frac{1}{r_1}\right] + \frac{1}{2\pi hL}\left[-\frac{1}{r^2}\right] = 0$$

$$= 0.$$

$$\frac{1}{\cancel{2\pi} k \cancel{L} \cancel{r}} = \frac{1}{\cancel{2\pi} h \cancel{L} r \cancel{r}}$$

$$\frac{1}{k} = \frac{1}{hr}.$$

$$k = hr$$

$$\frac{hr}{k} = 1$$

$$r = \frac{k}{h}.$$

$$\therefore \boxed{r_c = \frac{k}{h}.}$$

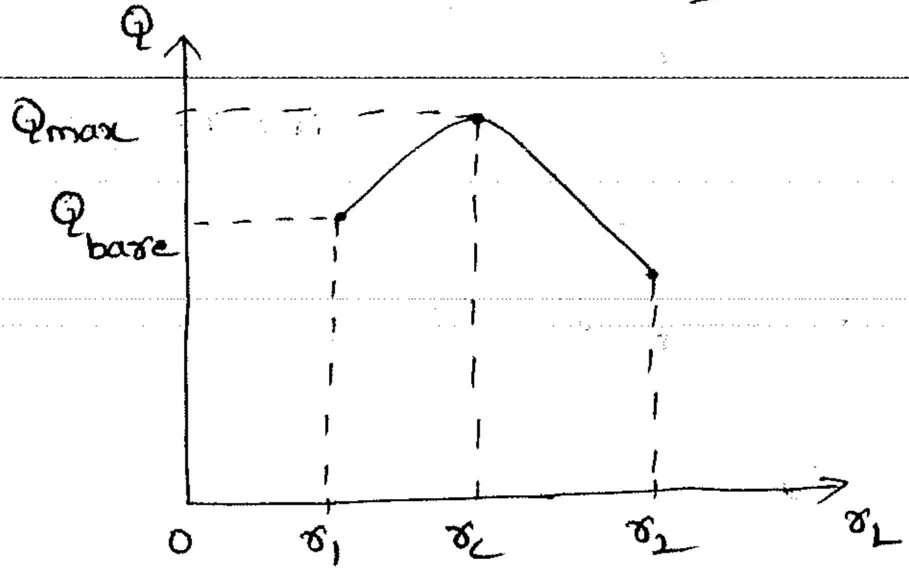
where r_c = critical radius of Insulation.

→ Adding of Insulation increase the heat loss upto a certain radious of cylinder that is called critical radious of Insulation. This thickness of Insulation layer is called critical thickness of Insulation, at which heat loss become maximum.

→ If critical thickness of Insulation is greater than the critical radius of r_c, any addition of Insulation on the pipe surface decrease the heat loss, if insulation thickness is smaller than r_c, the heat loss will increase as the in the case of electrical wires.

→ So keep our concentration about critical radius r_c, as per our requirement. If we want less HT Rate keep that $r_2 > r_c$. if we want more HT Rate keep that $r_1 < r_c$

→ That means, up to some extent HT Rate will increase, later increase the insulation, HT Rate ~~that~~ will decrease that converging and diverging meet point we called as r_c as shown in fig.



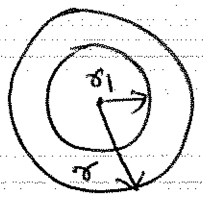
Effect of Insulation thickness on HT Rate

→ Adding of insulation inside on a pipe increasing Heat Transfer rate, so ~~that~~ in this case no concept of critical thickness of insulation

→ In case of Heat generation concepts like electrical wires, Heat dissipation must be more, more the current flow more the HT dissipation more the

$Q = I^2R \Rightarrow$ if $Q\uparrow \Rightarrow I$ increases square of value

3) Sphere :-



$$R_t = R_{cond} + R_{conv}$$

$$= \frac{r - r_1}{4\pi K r_1 r} + \frac{1}{h \cdot 4\pi r^2}$$

$$= \frac{1}{4\pi K}\left[\frac{1}{r_1} - \frac{1}{r}\right] + \frac{1}{4\pi h r^2}$$

for Max (or) Min, differentiate with $\underline{r}$ and equating to zero

$$\frac{dR_t}{dr} = \frac{1}{4\pi K}\left[0 - \left(\frac{-1}{r^2}\right)\right] + \frac{1}{4\pi}\left[\frac{-2}{r^3}\right] = 0$$

$$\frac{1}{\cancel{4\pi} K} * \frac{1}{\cancel{r^2}} = \frac{2}{\cancel{4\pi} h r^{\cancel{3}}}$$

$$\frac{1}{K} = \frac{2}{hr}$$

$$\frac{hr}{K} = 2$$

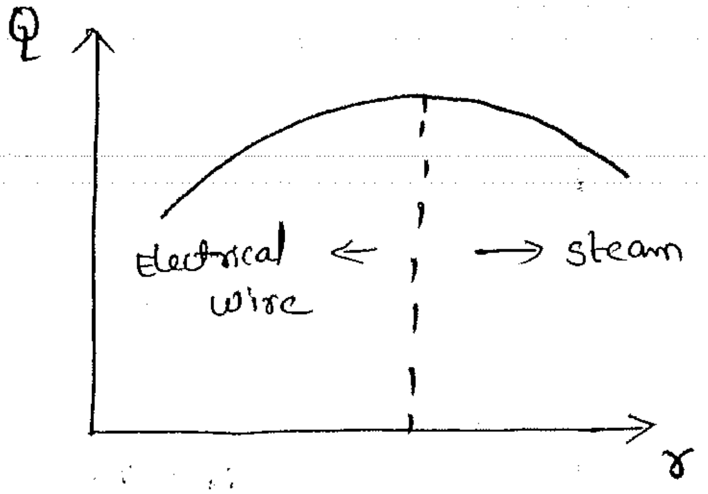
$$r = \frac{2K}{h}$$

$$\boxed{r_c = \frac{2K}{h}}$$

Note :-

1) In case of wires carrying current, there is a heat generation and for carrying current effectively this generated heat must be dissipated to surroundings, therefore ~~others~~ insulation ~~is~~ used in this case ~~heat~~ ~~transfer~~ transfer must ~~be~~ Less than the r_c

2) In case pipes carrying steam, insulation should be used in a such a manner that HT Rate is reduced, therefore insulation is must ~~be~~ greater than the r_c



3) so that we should check the r_c value by using above formulae

4) General values of thermal conductivity for insulating material is around 0.05 W/m-K and under natural convection with air $h = 5 W/m^2K$. Therefore the critical radius of insulation under this condition is $r_c = \frac{K}{h} = \frac{0.05}{5} = 0.01 m$ (or) 1 cm, as steam pipes radius is generally greater than the radius, therefore there is no need ~~of~~ to check the critical thickness ~~of~~ for steam pipe.

Conventional Questions :-

1) An Electrical wire, 2 mm in diameter is covered with a 2.5mm thick layer of plastic insulation ($k = 0.5\ W/mK$) to reduce the heat loss. Heat is dissipated from the outer surface of insulation to the surrounding air at 25°C by convection with heat transfer co-efficient of $10\ W/m^2K$. The wire is maintained at constant temperature of 120°C. Estimate the rate of heat dissipation from the wire per unit length with and with out insulation. calculate the thickness of insulation when the heat dissipation rate is maximum. what is Maximum value of heat dissipation.

Data

$$d_1 = 2mm = r_1 = 1mm$$

$$r_2 = 1mm + 2.5mm = 3.5mm$$

$$K_{ins} = 0.5\ W/mK.$$

$$T_\infty = 25^0$$

$$h = 10\ W/m^2K.$$

$$T_1 = T_S = 120^\circ C.$$

find

1) $Q = ?$ with out Insulation per meter length

2) $Q = ?$ with Insulion per meter length

Sol 1)

→ In 1st case what ever heat dissipation ie; pure ~~Conduction~~ Convection only

$$Q_1 = hA\Delta T$$

$$Q_1 = hA(T_s - T_\infty)$$

$$Q_1 = h \cdot \pi d_1 L (T_s - T_\infty)$$

$$\frac{Q_1}{L} = h \cdot \pi d_1 (T_s - T_\infty)$$

$$= 10 \times \pi \times 10^{-3} (120 - 25)$$

$$= 2.98 \text{ W/m length}$$

2) → In 2nd case Heat dissipation combined conduction and convection because of ~~addit~~ adding insulation

$$\Delta T = QR$$

$$Q = \frac{\Delta T}{R_{eff}}$$

$$Q = \frac{T_s - T_\infty}{R_{cond} + R_{conv}}$$

$$Q = \frac{T_s - T_\infty}{\frac{\ln(r_2/r_1)}{2\pi k L} + \frac{1}{h \cdot 2\pi r_2 L}}$$

$$\frac{Q}{L} = \frac{120 - 25}{\frac{\ln(3.5/1)}{2\pi \times 0.5} + \frac{1}{10 \times 2 \times \pi \times 3.5 \times 10^{-3}}}$$

$$\frac{Q}{L} = 19.2 \text{ W/m length}$$

Conclusion :- Adding Insulation increase heat dissipation

2) A 3 mm diameter and 5m long electrical wire is tightly wrapped with a 2-mm thick plastic cover whose thermal conductivity is k = 0.15 W/m·K. Electrical measurements indicate that a current of 10A passes through the wire and there is a voltage drop of 8 V along the wire. If the Insulated wire is exposed to a medium at $T_\infty = 30°C$ with heat transfer co-efficient of $h = 12 W/m^2K$. Determine the temperature at the Interface of the wire and plastic cover in steady condition. Also determine whether doubling the thickness of the plastic cover will increase or decrease this interface temperature.

Data

$d_1 = 3mm \Rightarrow r_1 = 1.5mm$

$L = 5m$

$r_2 = 1.5mm + 2mm = 3.5mm$

$K_{ins} = 0.15 W/mK.$

$I = 10A, \quad V = 8V$

$T_\infty = 30°C, \quad h = 12 W/m^2K$

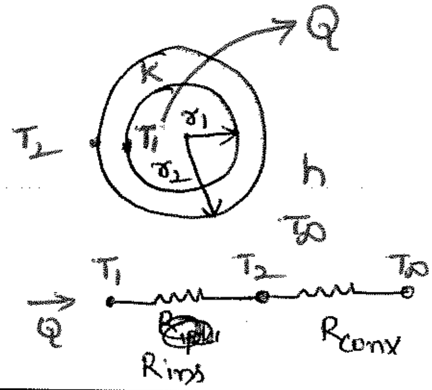
Find

1) Temperature Interface $(T_1) = ?$

2) What will happen HT if doubling the thickness

Sol

$\Delta T = Q R_{eff}$



$Q = VI$

$= 8 \times 10 = 80W.$

$R_{effe} = R_{Ins} + R_{conv}$

$$R_{Ins} = \frac{\ln(r_2/r_1)}{2\pi K L}$$

$$= \frac{\ln(3.5/1.5)}{2\times\pi\times 0.15\times 5}$$

$$= \cancel{0.76^\circ C/W}\ 0.18^\circ C/W.$$

$$R_{conv} = \frac{1}{hA_2}$$

$$= \frac{1}{h\,2\pi r_2 L}$$

$$= \frac{1}{12\times 2\times\pi\times 3.5\times 5}$$

$$= 0.76^\circ C/W.$$

$$\therefore R_{eff} = 0.18 + 0.76$$

$$= 0.94^\circ C/W.$$

$$\therefore \Delta T = Q R_{eff}$$

$$T_1 - T_\infty = Q R_{eff}$$

$$T_1 - 30 = 80 \times 0.94$$

$$\therefore T_1 = 105^\circ C.$$

2) ~~By Doubling~~ if Doubling the insulation

Critical radius $r_c = \frac{K}{h} = \frac{0.15}{12} = 0.0125 m$
$= 12.5 mm.$

if doubling insulation means $r_2 = 1.5mm + 4mm = 5.5mm$
$\therefore r_2 < r_c$, ; by doubling also Increase the Heat transfer.

3) A copper pipe carrying the Refrigerent at $-20^\circ C$ is 10mm in outer diameter and is exposed to ambient at $25^\circ C$. with convective coefficient of $50 W/m^2K$. It is proposed to apply the insulation of material having thermal conductivity of $0.5 W/mK$. Determine the thickness beyond which the heat gain will be reduced. Calculate the heat losses for 2.5mm, 7.5mm and 15mm thick layer of insulation over 1m length.

Data

$d = 10mm$, $r_1 = 5mm = 5\times10^{-3}m$

$T_1 = -20^\circ C$, $T_\infty = 25^\circ C$.

$h = 50 W/m^2K$

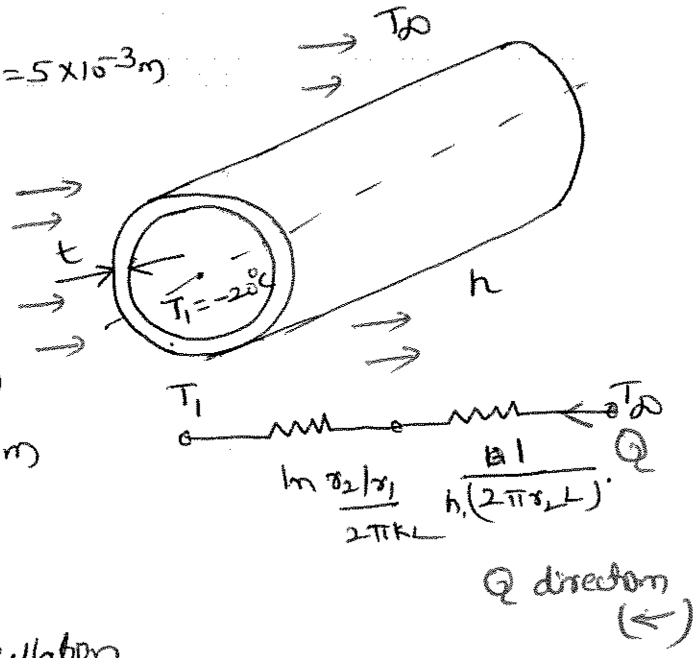
$K_{ins} = 0.5 W/mK$

$t_1 = 2.5mm = 2.5\times10^{-3}m$

$t_2 = 7.5mm = 7.5\times10^{-3}m$

$t_3 = 15mm = 15\times10^{-3}$

$L = 1m$



Find

1) critical thickness of Insulation

2) Heat loss for different thicknesses

Sol

1) critical radius of Insulation

$$r_{cr} = \frac{K}{h} = \frac{0.5}{50} = 0.01m = 10mm$$

$\therefore$ critical thickness of Insulation $(t_{cr}) = r_{cr} - r_1$

$= 10 - 5$

$= \underline{\underline{5mm}}$

2) Heat losses for different thickness

i) for 2.5mm thick

$$r_2 = r_1 + t_1 = \cancel{2.5}^{5} + 2.5 = 7.5mm = 7.5 \times 10^{-3} m$$

$$\Delta T = QR$$

$$Q = \frac{\Delta T}{R_{effetive}} = \frac{\Delta T}{R_{cond} + R_{conv}}$$

$$Q = \frac{\Delta T}{\frac{\ln(r_2/r_1)}{2\pi kL} + \frac{1}{(2\pi r_2 L)h}}$$

$$Q = \frac{25 - (-20)}{\frac{\ln(7.5/5)}{2\pi \times 0.5 \times 1} + \frac{1}{(2\pi \times 7.5 \times 10^{-3} \times 1) \times 50}}$$

$$\therefore Q = 81.3 W$$

ii) for 7.5mm thick

$$r_2 = r_1 + t_2 = 5 + 7.5 = 12.5mm = 12.5 \times 10^{-3} m$$

$$Q = \frac{25 - (-20)}{\frac{\ln(12.5/5)}{2\pi \times 0.5 \times 1} + \frac{1}{(2\pi \times 12.5 \times 10^{-3} \times 1) \times 50}}$$

$$= 82.37 W$$

iii) for 15mm thick

$$r_2 = r_1 + t_3 = 5 + 15 = 20mm = 20 \times 10^{-3} m$$

$$Q = \frac{25 - (-20)}{\frac{\ln(20/5)}{2\pi \times 0.5 \times 1} + \frac{1}{(2\pi \times 20 \times 10^{-3} \times 1) \times 50}}$$

$$Q = 74.95 W$$

Heat Transfer with Variable Thermal Conductivity

→ For most materials, the dependence of thermal conductivity on temparature is almost linear ie,..

$$\boxed{K = K_0(1+\beta T)}$$

where
K_0 = Thermal conductivity at 0°C temparature

β = const [depends on Material]

= Temparature co-efficient of thermal conductivity

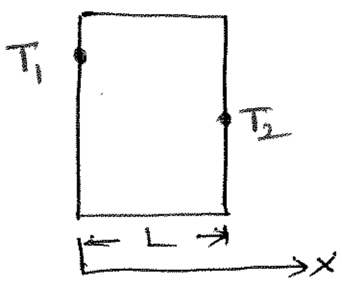
→ "β" may be positive (or) negitive depending up on whether thermal conductivity increases (or) decreases with temparature.

$\beta \rightarrow +ve$ for Non-Metals and insulation
[Exception of Magnesite Bricks]

$\beta \rightarrow -ve$ for metallic conductors
[Exception of Al & certain non-Fe Alloys]

1) plane slab :-

Assumptions :
1) steady state heat transfer
2) one Dimensional Heat transfer
3) Homogenious material
4) No Heat Generation



$$K = K_0(1+\beta T)$$

$$K_m = K_0(1+\beta T_m)$$

$$K_m = K_0\left(1+\beta\left(\frac{T_1+T_2}{2}\right)\right)$$

$\because T_m = T_{avg} = T_1+T_2/2$

∴ Heat conduction Equation for plane slab

$$Q = -KA \cdot \frac{dT}{dx}$$

$$\int_0^L \frac{Q\,dx}{A} = \int_{T_1}^{T_2} -K \cdot dT$$

$$\frac{Q}{A}\int_0^L dx = -\int_{T_1}^{T_2} K_0(1+\beta T)\,dT$$

$$\frac{Q}{A}(L)_0^L = -K_0\left[T + \frac{\beta T^2}{2}\right]_{T_1}^{T_2}$$

$$\frac{QL}{A} = -K_0\left[(T_2-T_1) + \frac{\beta}{2}(T_2^2-T_1^2)\right]$$

$$\frac{QL}{A} = K_0\left[(T_1-T_2) + \frac{\beta}{2}(T_1^2-T_2^2)\right]$$

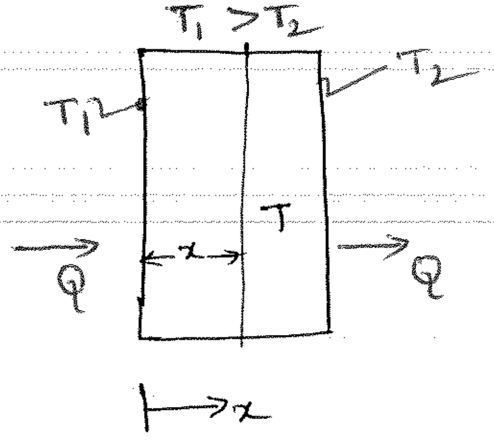
$$\frac{QL}{A} = (T_1-T_2)\underbrace{K_0\left[1+\beta\left(\frac{T_1+T_2}{2}\right)\right]}_{K_m}$$

$$\frac{QL}{A} = (T_1-T_2)K_m.$$

$$\boxed{\therefore Q = \frac{K_m A (T_1-T_2)}{L}}$$

While calculating Heat transfer use this equation with variation thermal conductivity with variation of temp. other wise use 'K' with replacing the "K_m" for @ const thermal conductivity

Temparature at any distance 'x' from the Left face :-



$$Q = -KA \cdot \frac{dT}{dx}$$

$$\int_0^x \frac{Q \cdot dx}{A} = \int_{T_1}^{T} -K\,dT$$

$$\frac{Q}{A}\int_0^x dx = -\int_{T_1}^{T} K_0(1+\beta T)\,dT$$

$$\frac{Q}{A}\Big[x\Big]_0^x = -K_0\left[T + \frac{\beta T^2}{2}\right]_{T_1}^{T}$$

$$\frac{Qx}{A} = -K_0\left((T-T_1) + \frac{\beta}{2}(T^2-T_1^2)\right)$$

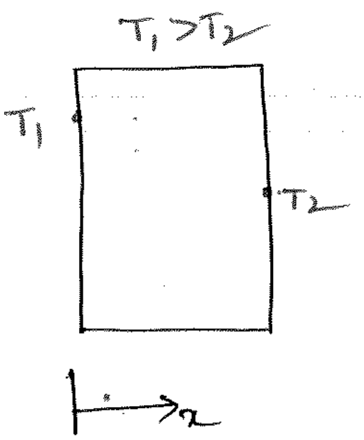
$x = T^2$
↓
not linear

$x = T$
↳ linear

$$\boxed{\frac{Qx}{A} = K_0\left[(T_1-T) + \frac{\beta}{2}(T_1^2-T^2)\right]}$$

The temp Profile :-

∴ The temp profile is not linear



$$K = K_0(1+\beta T)$$

differentiating

$$\frac{dK}{dx} = K_0\left(0 + \beta\frac{dT}{dx}\right)$$

$$\frac{dK}{dx} = K_0\,\beta\,\frac{dT}{dx}$$

K_0 ↳ +ve β ↳ ∓ve

Case 1 :- $\beta = 0$ [~~[illegible]~~]

$$K = K_0 (1 + \beta T)$$

$$= K_0 (1 + 0(T))$$

$$K = K_0 = \text{Constant}$$

∴ The ~~temparature~~ Thermal conductivity does not vary with temparature and equals to the constant value K_0

→ for steady state heat conduction

$$\phi = -KA \cdot \frac{dT}{dx} \Rightarrow \frac{\phi}{A} = -\left[K \cdot \frac{dT}{dx}\right]$$

the infinite wall $\frac{\phi}{A}$ is constant, so ~~[illegible]~~ $\left[K \cdot \frac{dT}{dx}\right]$ is the parameter

∴ $K \cdot \frac{dT}{dx}$ is constant, then the $\frac{dT}{dx}$ must be constant ∴ the slope of temp curve is constant and temparature profile is linear

Case 2 :- $\beta > 0$ [+ve]

$$K = K_0 (1 + \beta T)$$

$$\therefore \text{ie } K_0 \propto T$$

→ The Thermal conductivity of the wall material is directly proportional to the temp

→ if T ↑ ⇒ K ↑

→ if T ↓ ⇒ K ↓

→ Temp ↑ in x-direction ⇒ K ↓

→ Accordingly to maintain the parameter $\left(K \cdot \frac{dT}{dx}\right)$ constant, the term dT/dx must increase. Consequently the value of slope increases. Evidently with +ve value of β, the temp variation curve is of convex nature.

Exp:-

$$\text{if } x \uparrow \Rightarrow T \downarrow \Rightarrow K \downarrow \quad [\because K \alpha T]$$

$$\therefore \text{ for } K \cdot \frac{dT}{dx} = \text{const} \Rightarrow \frac{dT}{dx} \uparrow$$

$K \cdot \frac{dT}{dx}$

$3 \times 2 = 6$

$2 \times 3 = 6$

Case 3:- $\beta < 0$ [-ve]

$$K = K_0 (1 - \beta T)$$

$$\text{ie } K_0 \propto \frac{1}{T}$$

→ The thermal conductivity of wall material is inversely proportional to the temp

→ if $T \uparrow \Rightarrow K \downarrow$

→ if $T \downarrow \Rightarrow K \uparrow$

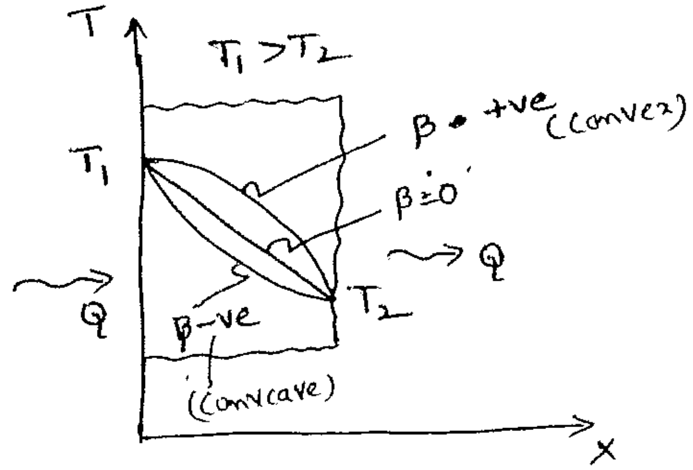
→ Temp $\uparrow$ in x-direction $\Rightarrow K \downarrow$

→ Accordingly to maintain the parameter $\left(K \cdot \frac{dT}{dx}\right)$ constant, the term $\frac{dT}{dx}$ must decreases. Consequently the value of slope decreases. Evidently with -ve value of β, the temp variation curve is concave nature.

$$\text{if } x \uparrow \Rightarrow T \downarrow \Rightarrow K \uparrow \quad \left[\because K \alpha \frac{1}{T}\right]$$

$$\text{for } K \cdot \frac{dT}{dx} = \text{const} \Rightarrow \frac{dT}{dx} \downarrow.$$

Temperature profile with variable 'K' :-



Conventional Questions :-

1) ~~A plane wall of~~ brick of 25cm thickness has temp of 1350°C and 50°C on its two sides, K of brick is $0.838(1+0.0007T)$ W/m°C, where T is in °C, calculate

i) Rate of Heat transfer

ii) temp at the Mid plane

iii) sketch the temp distribution

Data

$$L = x = 25\,cm = 0.25\,m$$

$$T_1 = 1350°C, \quad T_2 = 50°C$$

$$K = 0.838(1 + 0.0007T)\ W/m°C$$

Find

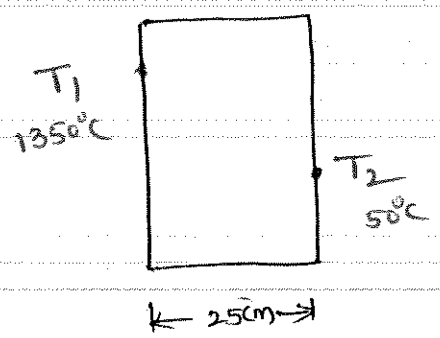
i) $\dot{Q} = ?$

2) $T_{mid} = T_{0.125m} = ?$

3) Temp profile

Sol

i) $$\dot{Q} = \frac{K_m A (T_1 - T_2)}{L}$$



$\Rightarrow K_{brick} = 0.838(1 + 0.0007T)$ ↗ β +ve

$$K_m = 0.838(1 + 0.0007 T_m)$$

$$\because T_m = \frac{T_1 + T_2}{2}$$

$$\therefore K_m = 0.838\left(1 + 0.0007\left(\frac{1350+50}{2}\right)\right)$$

$$K_m = 1.248 \ \text{W/m°C}$$

$$\therefore Q = \frac{K_m A (T_1 - T_2)}{L}$$

$$= \frac{1.248 \times 1 (1350 - 50)}{0.25}$$

$$Q = 6492 \ \text{W/m}^2$$

ii) Temp at Mid plane ($x = 0.25/2 = 0.125$m)

$$Q = -KA \cdot \frac{dT}{dx}$$

$$\int_0^{0.125} \frac{Q \cdot dx}{A} = \int_{1350}^{T} -K\,dT$$

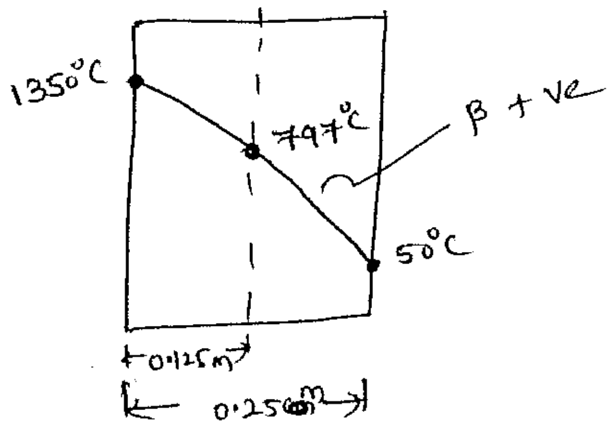
$$\frac{Q}{A}\int_0^{0.125} dx \quad \cancel{-K\int_{1350}^{T} dT} \quad -\int_{1350}^{T} 0.838(1 + 0.0007T)\,dT$$

$$\frac{Q}{A}\left[x\right]_0^{0.125} = \cancel{-K[\,]} \ -\left[0.838T + 0.0007\frac{T^2}{2}\right]_{1350}^{T}$$

$$\frac{6492}{1} \times 0.125 = -\left[\cancel{0.838}(1350 - T) + \frac{0.0007}{2}(1350^2 - T^2)\right]$$

$$\therefore T = 797°C$$

iii) Temp profile



2) Consider a 2m high and 0.7m wide bronze plate whose thickness is 0.1m. One side of the plate is maintained at a constant temp of 600K while the other side is maintained at 400K. The thermal conductivity of the bronze plate can be assumed to vary linearly in that temp range as $K(T) = k_0(1+\beta T)$, where $k_0 = 38\ W/mK$ and $\beta = 9.21 \times 10^{-4} K^{-1}$. Disregarding the edge effects and assuming steady one dimensional heat transfer. Determine the rate of heat conduction through the plate.

Data

$H = 2m,\ W = 0.7m,\quad L = x = 0.1m$

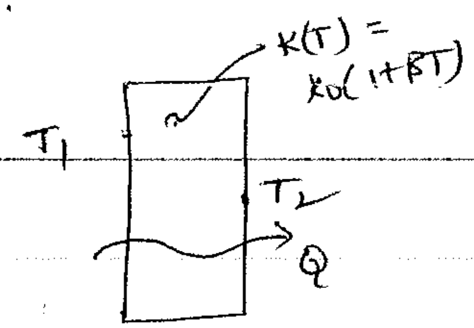
$T_1 = 600K\quad ,T_2 = 400K$

$K(T) = k_0(1+\beta T)$

$k_0 = 38\ W/mK\quad ,\beta = 9.21\times10^{-4}\ K^{-1}$

Find

1) Rate of Heat conduction $(\dot{Q}_{cond}) = ?$



Sol

$$\dot{Q} = \frac{K_m A (T_1 - T_2)}{L}$$

$$K_m = K_0 (1 + \beta T_m)$$

$$= K_0 \left(1 + \beta \left(\frac{T_1 + T_2}{2} \right) \right)$$

$$= 38 \left[1 + (9.21 \times 10^{-4}) \left(\frac{600 + 400}{2} \right) \right]$$

$$= 55.5 \, W/mK$$

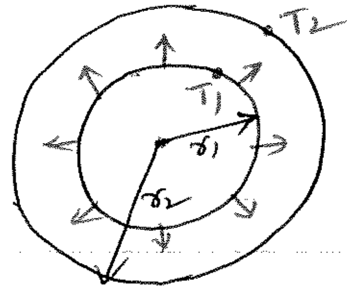
$$\therefore \dot{Q} = \frac{55.5 \times (0.2 \times 0.7)(600 - 400)}{0.1}$$

$$\dot{Q} = 155 \, KW$$

2) cylinder :-

Assumptions :-

1) steady state heat conduction
2) 1-D Heat transfer
3) No heat generation
4) Homogenious material



$K = K_0 (1 + \beta T)$

$$Q = -KA \frac{dT}{dr}$$

$$Q = -K \cdot 2\pi r L \frac{dT}{dr}$$

$$\int_{r_1}^{r_2} \frac{Q}{r} dr = \int_{T_1}^{T_2} -2\pi L K_0 (1 + \beta T) \, dT$$

$$Q \ln \frac{r_2}{r_1} = -2\pi K_0 L \left[T_1 + \frac{\beta T^2}{2} \right]_{T_1}^{T_2}$$

$$Q \ln \frac{r_2}{r_1} = -2\pi k_0 L\left[(T_1 - T_2) + \frac{\beta}{2}(T_1^2 - T_2^2)\right]$$

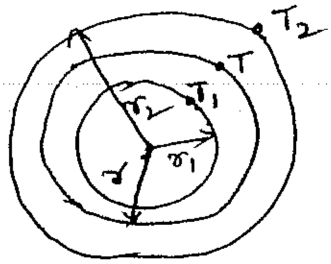
$$Q = \frac{2\pi k_0 L (T_1 - T_2)\left[1 + \frac{\beta}{2}(T_1 + T_2)\right]}{\ln \frac{r_2}{r_1}}$$

$$\boxed{Q = \frac{2\pi K_m L\,(T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)}}$$

$\because K_m = K_0\left(1 + \frac{\beta}{2}(T_1 + T_2)\right)$

Temperature at any radius 'r' :-

$$Q = -KA\frac{dT}{dr}$$



$$Q = -K \cdot 2\pi r L \frac{dT}{dr}$$

$K = K_0(1 + \beta T)$

$$\int_{r_1}^{r} \frac{Q}{r} dr = \int_{T_1}^{T} -2\pi L K\, dT$$

$$\int_{r_1}^{r} \frac{Q}{r} \cdot dr = \int_{T_1}^{T} -2\pi L K_0 (1 + \beta T) \cdot dT$$

$$\int_{r_1}^{r} \frac{Q}{r} dr = -2\pi L K_0 \int_{T_1}^{T} (1 + \beta T)\, dT$$

$$Q \ln\left[\frac{r}{r_1}\right] = -2\pi L K_0 \left[T + \frac{\beta}{2} T^2\right]_{T_1}^{T}$$

$$Q \ln\left[\frac{r_2}{r_1}\right] = -2\pi L K_0\left((T-T_1) + \frac{\beta}{2}(T^2-T_1^2)\right)$$

$$Q \ln\left[\frac{r}{r_1}\right] = 2\pi L K_0\left[(T_1-T) + \frac{\beta}{2}(T_1^2-T^2)\right]$$

$$Q \ln\left[\frac{r}{r_1}\right] = 2\pi L K_0 (T_1-T)\left[1+\frac{\beta}{2}(T_1+T_1)\right]$$

$$Q \ln\left[\frac{r}{r_1}\right] = 2\pi K_m L (T_1-T)$$

$$\boxed{Q = \frac{2\pi K_m L (T_1-T)}{\ln\left(\frac{r}{r_1}\right)}}$$

$\because K_m = K_0\left(1+\frac{\beta}{2}(T-T)\right)$

Conventional questions :-

1) The Inner and outer radia of hollow cylinder are 5cm & 10cm respectively. The Inside surface is maintained at 300°C and outside surface is maintained at 100°C, The Thermal conductivity varies with temp as $K = 0.5(1+10^{-3}T)$, where T is °C & K is in W/m-K. Determine

1) Heat transfer /m length of cylinder
2) Temp at mid thickness

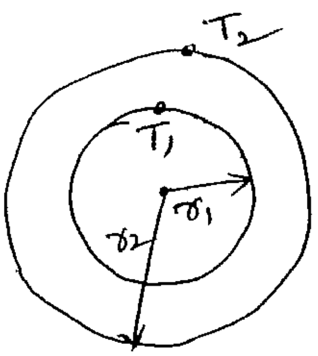
Data $r_1 = 5cm$, $r_2 = 10cm$

$T_1 = 300°C$, $T_2 = 100°C$

$K = 0.5(1+10^{-3}T)$

Find,
1) Q for meter length = ?
2) T_{mid} = ?

1)



$$Q = \frac{2\pi K_m L (T_1 - T_2)}{\ln\left[\frac{r_2}{r_1}\right]}$$

$$\because \quad K = \underset{\uparrow K_0}{0.5}(1 + \underset{\uparrow \beta}{10^{-3}} T)$$

$$K_m = K_0(1 + \beta T_m)$$

$$= K_0\left(1 + \beta\left(\frac{T_1 + T_2}{2}\right)\right)$$

$$= 0.5\left(1 + 10^{-3}\left(\frac{300 + 100}{2}\right)\right)$$

$$K_m = 0.6$$

$$Q = \frac{2 \times \pi \times 0.6 \times 1 (300 - 100)}{\ln\left(\frac{10}{5}\right)}$$

$$Q = 1087.7 \text{ w/m}$$

2)

$$Q = \frac{2\pi K_m L (T_1 - T_0)}{\ln\left(\frac{r}{r_1}\right)}$$

$K_m = K_0(1 + \beta T_m)$
$= 0.5\left(1 + 10^{-3}\left(\frac{300 + T}{2}\right)\right)$
substitute K_m and by solving

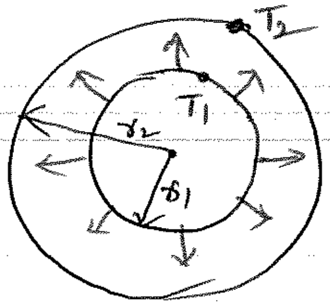
$$1087.7 = \frac{2 \times \pi \times 0.6 \times 1 (300 - T)}{\ln\left(\frac{7.5}{5}\right)}$$

$$T_0 = T_{mid} = 183^\circ C \quad 187^\circ C.$$

3) Sphere :-

Assumptions :-

1) Steady state
2) 1-Dimensional
3) No Heat generation
4) Homogeneous material



$k = k_0(1+\beta T)$

$$Q = -KA \cdot \frac{dT}{dr}$$

$$Q = -K\,4\pi r^2 \frac{dT}{dr}$$

$$Q \cdot \frac{dr}{r^2} = -4\pi K\, dT$$

$$\int_{r_1}^{r_2} \frac{Q\, dr}{r^2} = \int_{T_1}^{T_2} -4\pi K_0 (1+\beta T) \cdot dT$$

$$Q \int_{r_1}^{r_2} \frac{1}{r^2}\, dr = -4\pi K_0 \int_{T_1}^{T_2} (1+\beta T) \cdot dT$$

$$Q \left[-\frac{1}{r}\right]_{r_1}^{r_2} = -4\pi K_0 \left(T + \frac{\beta}{2} T\right)_{T_1}^{T_2}$$

$$Q \left[-\frac{1}{r_2} + \frac{1}{r_1}\right] = -4\pi K_0 \left[(T_2 - T_1) + \frac{\beta}{2}(T_2^2 - T_1^2)\right]$$

$$Q \left[\frac{-r_1 + r_2}{r_1 r_2}\right] = 4\pi K_0 \left[(T_1 - T_2) + \frac{\beta}{2}(T_1^2 - T_2^2)\right]$$

$$Q\left[\frac{r_2-r_1}{r_1 r_2}\right] = 4\pi(T_1-T_2)\,k_0\left[1+\beta\left(\frac{T_1+T_2}{2}\right)\right]$$

$$Q\left[\frac{r_2-r_1}{r_1 r_2}\right] = 4\pi(T_1-T_2)\,K_m$$

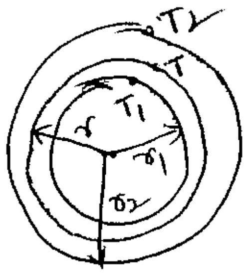
$$Q\left[\frac{r_2-r_1}{r_1 r_2}\right] = 4\pi K_m (T_1-T_2)$$

$$\boxed{Q = \frac{4\pi K_m r_1 r_2 (T_1-T_2)}{r_2-r_1}}$$

<u>Temparature at any radius "r" :-</u>

$$K = k_0(1+\beta T)$$

$$K_m = k_0\left(1+\beta\left(\frac{T_1+T_2}{2}\right)\right)$$



$$Q = -KA\frac{dT}{dr}$$

similary above Equation limits are r_1 & r, T_1 & T, we get

$$\boxed{Q = \frac{4\pi r_1 r\, K_m (T_1-T)}{r-r_1}}$$

Conventional questions:-

1) A spherical shaped container of inner radius 15cm stores a cryogenic substance at -180°C. The inflow of heat is checked by covering it with 10cm thick insulation material which has thermal conductivity prescribed by the relation $K = 0.03(1+0.005T)$ where "T" is in °C and K is in W/mK. If the outer surface of insulation layer is at 15°C. Determine the heat inflow, the temperature at mid radius and the radius at which the temp is -40°C

Data

$r_1 = 15cm = 0.15m$ $\quad T_1 = -180°C$ $\quad r_2 = 15+10 = 25cm = 0.25m$

$$K = \underset{\uparrow k_0}{0.03}(1+\underset{\uparrow \beta}{0.005}T)$$

$$T_2 = 15°C.$$

find

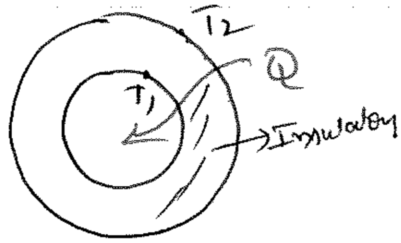
1) heat inflow = ?

2) T_{mid} = ?

3) r = ? at $T = -40°C$.

Sol

1) $$Q = \frac{4\pi k_m r_1 r_2 (T_1 - T_2)}{r_2 - r_1}$$



Heat inflow means, heat flow radially inward

$$K_m = k_0(1+\beta T_m)$$

$$= 0.03\left(1+\beta\left(\frac{T_1+T_2}{2}\right)\right)$$

$$= 0.03\left(1+\beta\left(\frac{15+(-180)}{2}\right)\right)$$

$$= 0.0176\, W/mK$$

T_2 = in
T_1 = out

$$\therefore Q = \frac{4\times\pi\times0.0176\times0.15\times0.25(15-(-180))}{0.25-0.15}$$

$$Q = \underline{15.75W}$$

2) $r_{mid} = r_1 + \frac{10}{2} = 15+5 = 20cm = 0.2m$

$$Q = \frac{4\pi K_m r_1 r_{mid}(T_1 - T)}{r - r_1}$$

$$K_m = K_0(1+\beta T_m)$$

$$= K_0\left(1+\beta\left(\frac{T_1+T}{2}\right)\right)$$

$$= 0.03\left(1+0.005\left(\frac{T+(-180)}{2}\right)\right)$$

$$= 0.0165 + 7.5\times10^{-5}T$$

$$15.75 = \frac{4\pi(0.0165+7.5\times10^{-5}T)\times0.15\times0.2(T-(-180))}{0.25-0.2}$$

$$= 7.536(0.0165+7.5\times10^{-5})\times(T+180)$$

$$= 7.536[0.0165T + 7.5\times10^{-5}T^2 + 2.97 + 0.0135T]$$

$$= 56.52\times10^{-5}T^2 + 0.2262T + 22.38$$

$$= 56.52\times10^{-5}T^2 + 0.2262T + 6.63 = 0$$

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$T = \frac{-0.2262 \pm \sqrt{(0.2262)^2 - 4(56.25\times10^{-5})\times6.63)}}{2\times56.25\times10^{-5}}$$

$$T = -32.44^\circ C \text{ (or) } -369.33^\circ C$$

∴ The Acceptable Answer is must be $-180°C$ to $15°C$

therefore. The Mid temparature is

$$T_{mid} = -32.44°C$$

3) $r = ?$ at $T = -40°C$

$$Q = \frac{4\pi K_m r_1 r \Delta T}{r - r_1}$$

$$= \frac{4\pi K_m \Delta T}{\left(\frac{1}{r_1} - \frac{1}{r}\right)}$$

$$K_m = 0.03\left[1 + 0.005 \frac{-40 + (-180)}{2}\right]$$

$$= 0.01275 \text{ w/mk}$$

$$\therefore 15.75 = \frac{4 \times \pi \times 0.01275 (-40 - (-180))}{\frac{1}{0.15} - \frac{1}{r}}$$

$\cancel{\frac{1}{r} = 6.67}$ $\quad$ ~~r = 5.145 m~~

$$r = 0.194 \text{ m}$$